https://orcid.org/0000-0003-4395-9281

UDC 334.7

M.E. ALIYEV*

ONE APPROACH TO SOLVING THE PROBLEM OF PRICING IN THE RESIDENTIAL REAL ESTATE MARKET

*Institute of Control Systems of Azerbaijan National Academy of Science, Baku, Azerbaijan

Анотація. *Початком розвитку поняття «середнє значення» вважається етап, коли воно стало центральним членом безперервної пропорції. Однак визначення середнього значення як центральної величини прогресії не дає можливості вивести його значення по відношенню до послідовності n членів незалежно від їх порядку. З огляду на це, постає необхідність формального узагальнення середніх значень, що і зумовлює перехід від безперервних пропорцій до арифметичної, геометричної і гармонічної прогресій. У даний час середньому значенню надають статистичного значення, пов'язуючи його з економічними категоріями. У математиці в теорії середніх значень виокремлюються два види: власне середні значення і середні арифметичні. При цьому кожен із видів середніх значень може виступати у формі як простого, так і середньозваженого значення, вибір між якими зумовлюється матеріальною природою об'єкта дослідження. Формули простих середніх значень застосовуються в тих випадках, коли індивідуальні значення величин, що усереднюються, не повторюються. Якщо в розрахункових формулах середніх значень присутня певна частота повторень індивідуальних значень величин, то в цьому випадку формули простих середніх називаються формулами середньозважених. З огляду на це, у статті пропонується підхід до опису середніх соціально-економічних показників на прикладі показника «Середня ринкова вартість кв. метра житла» на ринку нерухомості в уявному регіоні шляхом усереднення цін на основі оцінки їх середньозважених значень для конкретного регіону. В рамках даного підходу ідентифікується функція розподілу ваги вартості житла як автомодельного рішення в диференціальному рівнянні з частинними похідними з початковими і крайовими умовами.*

Ключові слова: *усереднення, середнє значення, автомодельне рішення, середньозважене значення, середня вартість житла.*

Abstract. *A stage when an average value began to be considered as a central term of a continuous proportion is considered to be the beginning of the development of the «average value» concept. However, this concept, as the central value of the progression, does not make it possible to derive it in relation to a sequence of n terms, regardless of the order in which they follow each other. For this purpose, it is necessary to resort to a formal generalization of average values which predetermined the transition from continuous proportions to progressions – arithmetic, geometric and harmonic. Currently, a statistical value is given to the average value through its connection to economic categories. In the mathematical theory of average values, there are distinguished two types of them – average values themselves and arithmetic average values. At the same time, each type of average values can act either in the form of a simple or weighted average where the correct choice of the form of the average follows from the material nature of the object of study. Simple average formulas are used in cases where the individual values of the averaged feature do not repeat. If the frequency of repetition of individual values of a feature is present in the calculation formulas of averages, then in this case the formulas of simple averages are called as formulas of weighted averages. With the respect to it, the paper proposes an approach to the description of average socio-economic indicators, using the example of the indicator «Average market value of a square meter of housing» in the real estate market in a hypothetical region by averaging prices based on their weighted average assessment in a particular area. Within the framework of this approach, the distribution function of the weights of housing costs is identified as a self-similar solution (automodel) of a partial differential equation with initial and boundary conditions.*

Keywords: *averaging, mean value, self-similar solution, weighted average, average cost of housing.*

DOI: 10.34121/1028-9763-2021-4-46-57

1. Introduction

Averaging is a type of measurement that is considered to be one of the most common operations in data acquisition. Particularly in technical systems, achieving a required level of accuracy during averaging is usually attained through means of multiple measurements, wherein the results of individual measurements are partially compensated by positive and negative deviations from the exact value. The accuracy of their mutual compensation improves as the number of measurements increases because the average value of negative deviations modularly approaches the average value of positive deviations.

However, when considering statistical data samples that have the «historical» status, there arises an issue. As a rule, such samples characterize the behavior of economic systems over a certain period of time or represent sets of indicators values reflecting committed phenomena which, as of now, cannot be measured anew. Nevertheless, it is necessary to be able to properly analyze and adequately assess this variation of indicators for data acquisition. It should be mentioned that even samples reflecting economic processes and/or phenomena in the past, present and future are important for consideration.

The aim of the work is to suggest a new approach to the process of statistical averaging of statistical data and to test this approach on the process of pricing in the residential real estate market.

2. Main indicators of measurement results averaging

One of the most common operations performed in data acquisition and control systems is the averaging of multiple measurements results. Intuitively, it is clear that this process leads to increased accuracy, since the results of individual measurements have both positive and negative deviations from the exact value and, consequently, they are partially mutually compensated, as it was mentioned above. In practice, it is important to obtain a quantitative relation between the number of measurements and an error of the averaged result.

Let us consider some means of measurement, for example, an analogous input measuring module with the task of measuring and entering values of voltage x into a computer (see Fig. 1). In general cases, a sensor, a communication line between the sensor and a module, as well as the module itself, are all affected by electromagnetic interference and noises generated from operational amplifiers, analog-to-digital converter, resistors, microprocessor part of the module, etc. These interferences, which influence the measurement object, will not be considered, because they are not a part of the measurement channel.

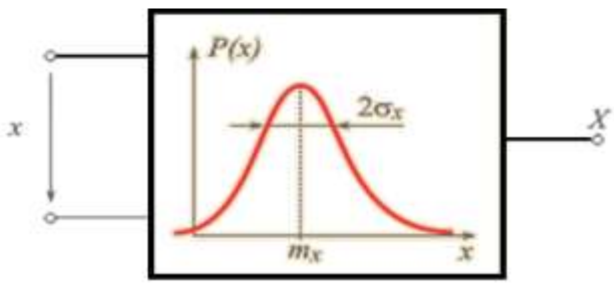

Figure 1 – The input module measures a physical quantity x and outputs a random quantity X

These reasons lead to the fact that the measurement result becomes a random variable, the value of which varies from measurement to measurement. A random quantity X can be described by a distribution function with mathematical expectation $M(X) = m_x$ and standard deviation σ_x which is taken as a random component of the measuring instrument error [1]. The variance of a random variable is as follows[1]:

$$D(X) = \sigma_x^2 . \tag{1}$$

The actual uncertainty of the measuring instrument is determined by the manufacturer and is usually specified in the operating documentation. The given error value includes both systematic and *random* components. If the random component exceeds 10% of the systematic component, it is indicated separately. In some cases, the random component is indicated by means of an autocorrelation function which spots the development of trends over time or influences power spectral density.

The random component of the given error can be reduced by averaging the results of multiple measurements. If the systematic component prevails in the error composition, averaging does not lead to an increase in accuracy. The presence of a random component can be judged from a scatter presentation of the results of single measurements.

Let us suppose that with the help of the measuring module there are performed N measurements, as a result of which the values $x_1, x_2, \dots, x_N$ are obtained. The measurement results are averaged using the arithmetic mean formula which goes as follows:

$$x_{\text{avr}} = \frac{1}{N}\sum_{k=1}^{N} x_k . \tag{2}$$

However, x_{avr} is also a random variable, because by performing several series of measurements and averaging each of them we will obtain different mean values of x_{avr} for each series. Despite that, x_{avr} will have less variance (standard deviation) than the measuring device. According to [3, 4], this process can be shown as follows.

Let us assume that the results of measurements $x_1, x_2, \dots, x_N$ are independent random variables. Then the variance of their arithmetic mean will be

$$D(x_{\text{avr}}) = D\left(\frac{1}{N}\sum_{k=1}^{N} x_k\right) = \frac{1}{N^2} D\left(\sum_{k=1}^{N} x_k\right) = \frac{1}{N^2} N D(x_k) = \frac{D(x_k)}{N} = \frac{\sigma_x^2}{N} \tag{3}$$

and from which subsequently follows

$$\sigma_{\text{avr}} = \frac{\sigma_x}{\sqrt{N}},$$

considering

$$\sigma_{\text{avr}} = \sqrt{D(x_{\text{avr}})} .$$

According to [4], the following two properties of the variance operator are used in the derivation of the formula (3):

• the variance of the product of a random variable and a constant is equal to the variance of the random variable multiplied by the square of the constant;

---

[1] The dispersion σ^2 is usually understood as the density of the distribution of neighboring elements relative to some middle x_0. According to [2], the dispersion is calculated as a standard deviation of elements x_k $(k = 1 \div n)$ from the center x_0 by formula $\sigma^2 = \frac{1}{n}\sum_{k=1}^{n}(x_k - x_0)^2$.

• the variance of the sum of random variables is equal to the sum of variances of each of them. In addition, it is assumed that all measurements are performed by the same instrument, i.e., the variance of all measurements is the same and equal to $D(xk) = \sigma x^2$, and that the random variables are known to be uncorrelated.

3. Averaging in the transfer pricing methodology

As a rule, representatives of tax and antimonopoly services of the state carry out verification of correctness of prices put out by companies for goods and services sold by them in the cases determined by the law. The legislation of many countries allows such an inspection if the cost of sold goods and services significantly deviates from the *average market price* [5]. Furthermore, if a company's pricing policy turns out to be incorrect, the tax authorities have every reason to charge additional taxes.

The reason for such actions of the public service lies in the fact that the pricing policy chosen in private companies significantly affects the amount of many direct and indirect payments to the budget, since the tax base is the company's revenue as a reflection of the value of sold products. In particular, if, as a result of the inspection, it is found that the prices differ, for example, by more than 20% from the prices for similar goods, then it follows that the tax authorities have the right to increase the tax base and impose a penalty. To ensure that the cost of sold goods and services does not raise questions from the tax authorities, there is used the transfer pricing methodology, the essence of which is described below.

One of the methods of transfer methodology is the calculation of the arithmetic average of the products pricing. It requires data on the cost of each of the goods (x_k), considered in the calculation and their quantity (N). In this case, the arithmetic mean formula for determining the cost of products looks like the aforementioned (1). This formula is used to determine the average cost of goods and services in certain groups that are identical and/or homogeneous to it. For instance, goods with almost identical basic characteristics are considered to be identical. Homogeneous products are products with similar parameters that are made of similar components and possess the same scope of application.

The arithmetic mean formula (1) is often used to compare pricing within one locality, region or a period of time. To study the pricing of goods and services to which there can be applied other indicators, such as different types of outlets and different sales periods, more complex calculations are often utilized. In particular, when studying the average market price, the weighted average formula is used, as follows [6–8]:

$$I_{\text{avr}} = \frac{\sum_{k=1}^{n} w_k \cdot I_k}{\sum_{k=1}^{n} w_k},$$

where I_{avr} is the weighted average indicator and I_k is the calculated k-th indicator. Additionally, w_k is the weighting coefficient (or, to put it simply, the weight) of the k-th indicator, also characterizing its distinctiveness and priority in one row with other indicators of the form I.

As a rule, the weighted average formula is used when there is data on sales of homogeneous products at different prices and in different batches. In addition, this system of calculating average price indicators of sold goods is used when they are accounted in quantitative terms, such as kilograms, grams, meters, liters, etc. According to [5, 6], the formula for calculating the weighted average cost of goods can be described as follows:

$$\bar{P} = \frac{\sum_{i=1}^{N} P_i \cdot Q_i}{\sum_{i=1}^{N} Q_i}, \tag{4}$$

where Q_i is the quantity of i-th good sold in natural units (meters, kilograms, liters, etc.), which in the formula acts as a weighting factor. As can be seen, to calculate the weighted average, first of all it is necessary to multiply the number of products sold in kind by the cost of a commodity unit in each countable batch. After that the result is divided by the total amount of products sold in kind. It must be remembered that the weighted average will not reflect the level of the usual cost of goods, i.e., it will differ markedly from the arithmetic average calculated by formulas such as (1).

In the process of pricing heterogeneous products of varying value, the average harmonic system is usually used. This approach is actually utilized quite rarely and usually only in cases where the amount purchased from different sellers is the same, while the total amount of purchased products is unknown. If we apply the arithmetic mean method for calculations with the same amount of input data, the final figure will be higher than the figure found using the harmonic mean. According to [5], the formula for determining the harmonic mean is as follows:

$$\bar{x}_{\text{harm}} = \frac{1+1+\ldots+1}{\frac{1}{x_1}+\frac{1}{x_2}+\ldots+\frac{1}{x_n}} = \frac{N}{\sum_{j=1}^{N}\frac{1}{x_j}}, \tag{5}$$

Where x_j $(j = 1 \div N)$ is the cost of goods purchased in different outlets.

The harmonic mean is used when the weights of the trait values are the same. Let's take the following example. Suppose it is necessary to calculate the average price of two goods P_{avr} bought from two different sellers (as such, $N = 2$) in the same quantity but at different prices: one buyer for \$100, and the other one for \$90. At the same time, the sellers have the same weighting coefficients. It means that they are identical in the buying and selling process, i.e., they do not have any advantage over one other. Applying the average harmonic method, the average price of the product is calculated by the formula

$$P_{\text{harm}} = \frac{1+1}{\frac{1}{100}+\frac{1}{90}} = \frac{2}{\frac{90+100}{9000}} = \frac{18000}{190} = 94{,}7 .$$

However, if it was necessary to calculate the arithmetic mean, it would actually sum up to \$95.

Obviously, to calculate the average harmonic index, the number of analyzed purchases should be firstly summed up. After that, it is necessary to divide the unit (the number of transactions with one seller) by the price at which the goods are purchased. Then the totals are summed up. As a result of dividing the total number of sale transactions by the already found amount, the required value of the average harmonic index is got.

4. The novel approach to achieving the necessary accuracy of averaging

Application of quantitative methods of analysis and processing of relevant information allows us to obtain the most accurate and adequate assessment of the activity of companies, micro- and macroeconomic indicators, as well as their future values. Therefore, in contrast to mechanistic systems, where multiple measurements of a particular indicator are possible, in the case of historical samples multiple refinements of its averaging are impossible. This is clearly due to the fact that the samples have been recorded at a moment in the past and are thus unchangeable. Actually, these are the prerequisites for the current research which is largely based on the results of the work [9].

Over the last few centuries, the efforts of many famous scientists, such as Laplace, Euler, Dalembert, Adamar, etc., formed a universal linear theory of processes in different environments, in which it is possible to distinguish basic models, reflecting a striking variety of phenomena of different natures [10]. In particular, these are findings such as the heat conduction equation (reflecting the distribution of heat in a limited area), the wave equation (describing, eponymously, the oscillation of a string) and the Laplace equation [10] (defining the field potential created by a system of electric charges).

Let us now consider an extended interval $[a, b]$ of length l which covers a set of data, each of which, in its own way, reflects the required value of the economic indicator I at a particular point in time. For the subsequent processing of values of such indicators, various methods of averaging are used in mathematical statistics. However, in this section, there is used another method.

For each datum from the interval $[a, b]$, the spatial coordinate u ($a \le u \le b$ or $0 \le u \le l$) should be compared. Suppose that at the initial stage at $t = 0$ the value of the index I can be reflected by the positive function $w(u,\underline{0}) = w_o(u)$ which determines the initial distribution of weights between the data u_k. There arises the question: how should the distribution of weights (or the function $w(u,t)$) change in order to improve the adequacy of the averaging of any indicator I ?

Firstly, based on the concept of weights, the desired function must take its values on the unit segment, i.e., $w(u,t^*) \in [0,1]$. Secondly, its largest value should be the one which implies that the average value of u_{avr} has the largest weight. This completes the process of searching for the function $w(u,t^*)$, and, as a consequence, the averaging of the index I can be established by the following weighted average formula

$$u_{\text{avr}} = \frac{\sum_{k=1}^{s} w(u_k,t^*)\cdot u_k}{\sum_{k=1}^{s} w(u_k,t^*)}, \tag{6}$$

where s is the number of measurements of the index I .

The answer to the stated above question will be searched for in a continuous medium[2] which in our case is characterized by continuous functions of spatial and temporal coordinates. To this end, let us define around the point $x \in [a,b]$ a neighborhood with the radius $\Delta u/2 : (x - \Delta u/2;\ x + \Delta u/2)$. Let $w(x,\tau)$ be the distribution function of weights at time τ. After a small time interval Δt, this function will be equal to $w(x,\ \tau + \Delta t)$. Evidently, it can change only due to the difference of the «fluxes» of positive (on the right) $D(x + \Delta u/2, \tau)$ and negative (on the left) $D(x - \Delta u/2, \tau)$ deviations from the value of x, or, precisely:

$$w(x,\tau + \Delta t) - w(x,\tau) = [D(x + \Delta u/2,\tau) - D(x - \Delta u/2,\tau)]\Delta t . \tag{7}$$

In fact, the equation (7) reflects the law of conservation of substances. At an intuitive level, it is clear that the greater the difference between the weights of the given x changing over a

[2] The existing idea of a continuous medium has become the key paradigm of modern natural science which was based, among other things, on the theory of heat conduction and new mathematical methods laid down by the French mathematician Fourier.

small-time interval Δt, the greater the value of the difference between positive and negative deviations. This means that it is necessary to find a more adequate averaging of the index I value.

Now let us move to the limit in a continuous medium, i.e., assuming

$$D(u,t) \cong [w(u+\Delta u/2,\ t) - w(u-\Delta u/2,\ t)/\Delta u], \tag{8}$$

let us assume that

$$D(u,t) \cong \frac{\partial w(u,t)}{\partial u}. \tag{9}$$

(9) is the definition of the partial derivative with respect to the variable u. (8) and (9) are two different interpretations of the same thing: (8) in terms of the difference, (9) in terms of the partial derivative. In other words, (8) and (9) are the same entry, the only difference is that the difference $\Delta u = (\Delta u/2) + (\Delta u/2)$ in equality (8) is equivalent to ∂u in equality (9). Note that in (8) $(u+\Delta u/2) - (u-\Delta u/2) = \Delta u \cong \partial u$.

Substituting expression (8) by the equation (7) and making the limit transition at $\Delta u \to 0$ and $\Delta t \to 0$, the following partial differential equation is obtained:

$$\frac{\partial w(u,t)}{\partial t} = \frac{\partial^2 w(u,t)}{\partial u^2} \tag{10}$$

with appropriate initial and boundary conditions

$$w(u,0) = w_0(u);\ \ w(0,t) = w(l,t) = 0. \tag{11}$$

Equation (10) is analogous to the thermal conductivity equation, describing the transfer of heat at a single value of the diffusivity coefficient in the right part of the differential equation. In some sources [11], this coefficient is called the heat transfer coefficient, although the diffusivity coefficient is calculated as the ratio

$$a = \lambda/(pc),$$

where λ is the thermal conductivity coefficient, ρ is the density and c is the heat capacity.

In addition, the equation (10) with the appropriate coefficient in the right part of the differential equation is used to describe diffusion of particles, penetration of a magnetic field into plasma and many other processes likewise.

To solve the problem (10)–(11), it is necessary to set in advance the initial state of the desired function $w(u,0) = w_0(u)$. The presence of boundary conditions $w(0,t) = w(l,t) = 0$ means that on the boundary of the interval $[0, l]$ zero values of weights are maintained. Under initial and boundary conditions, the computer stepwise solution of the problem (10)–(11) looks as shown in Fig. 2. The initial solution $w(u,0) = w_0(u)$ is rather a «narrow» symmetric curve with maximum amplitude (or vertex) in the point with abscissa – the arithmetic mean of all measurements of the considered index which in the context of problem (10)–(11) is the middle of the interval $[0, l]$.

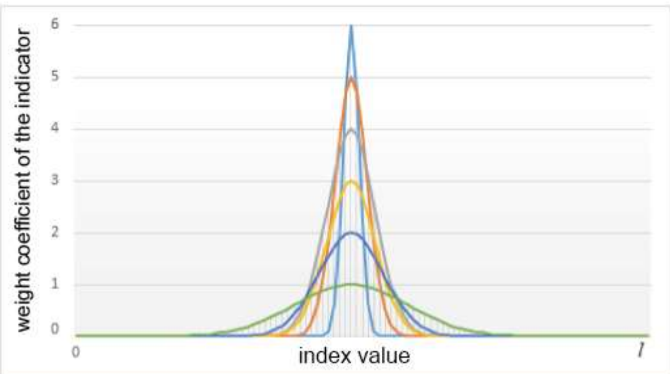

Figure 2 – Evolution of the indicator weights distribution

Let us trace the evolution of the weight distribution curve. Fig. 2 shows the profiles (external outlines) of weight distribution functions at different steps of the iterative (i.e., repeated) search for the desired solution of problem (10)–(11). It can be seen that the maximum amplitude A of the curve $w(u,t)$ decreases and the width of the profile L at the level $A/2$ grows. Thus, based on the stated above reasoning and assuming that the sum of weights in (6) remains unchanged, it becomes obvious that $A \cdot L \cong const$.

According to [10, 12], the desired function that depends on two variables can be found as a product of two functions, for example, in the form of

$$w(u,t) = f(u) \cdot g(t), \tag{14}$$

where in the process of evolution the function $f(u)$ determines a spatial configuration (a solution form) of $w(u,t)$, and $g(t)$ demonstrates changes in the amplitude of the distribution function of the indicator weights. Solutions of this kind do not change their forms in the process of evolution (or stepwise iteration); they are called self-similar, i.e., in this case functions of the formula (14) play a significant role in the study of nonlinear processes and phenomena[3].

Suppose that an automodel solution of problem (10)–(11) is found in (14). Then $f(u)$ is called an eigenfunction which most closely depends on the boundary conditions. In particular, according to [10, 12] in the self-similar solution of problem (10)–(11) there is an uncountable number of functions of the formula

$$f_k(u) = \sin[\pi k u / l], \quad k = 1, 2, \ldots .$$

At the same time, each of them corresponds to a different function of amplitude change over time

$$g_k(t) = (A - t) \cdot \exp[-(\pi k / l)^2 \cdot t], \; k = 1, 2, \; \ldots ,$$

where A is the value of the maximum amplitude.

5. Pricing in the residential real estate market by identification of the weight distribution function of the indicator measurements

Let us consider the task of determining the value of the economic indicator «Average market price per square meter of housing» for a hypothetical region. It is obvious that there is a large number of both objective and subjective factors that have a significant impact on pricing in this sector of the economy.

Suppose that in this region the cost per square meter of housing in each of the five peripheral districts is \$800 and in the central district – \$2000. Then, according to the traditional method of calculation, the average cost per square meter of housing in the region as a whole will be the arithmetic mean, calculated according to the formula (1), i.e., \$1000. It is obvious that this value does not quite adequately reflect the real state of the average price level per square meter of housing in the region, because it does not take into account the relative influence of all existing factors.

To apply the weighted summation formula (6), we identify an indicator of the weight distribution function corresponding to this problem as a solution of problem (10)–(11). To this end, let us consider equation (10) on the interval [0, 2050] that covers a statistical sample of data on the market price per square meter of housing with the following initial and boundary conditions:

$$w(u, 0) = w_0(u), \; w(0, t) = w\left(2050, t\right).$$

---

[3] A family of sinusoidal functions, fragments of which are shown in Fig. 3 and Fig. 4, is used in this work as automodel functions. An automodel function or self-similar solution is a solution to some equation of two independent variables, in which the independent variables, in our case u and t, do not appear in an arbitrary way but only in the combination [13].

We will search for the required function in the form of automodel solution (14) where as an initial condition at $t_0 = 0$ we choose the function

$$w(u, t_0) = 7\sin[\pi u / 2050]\exp[-(\pi / 2050)^2 t_0] = 7\sin[\pi u / 2050].$$

The computer solution, which is shown in Fig. 3, gave the final solution to the problem at $t = 6$ in the form of the following weight distribution function:

$$w(u, 6) = \sin[\pi u / 2050] \cdot \exp[-(\pi / 2050)^2 \cdot 6], \qquad (15)$$

the largest value of which is 1. In this case the weights in the housing pricing of both types of neighborhoods will be the corresponding numbers: *w*(800, 6)=0,9411 and *w*(2000, 6)=0,0765.

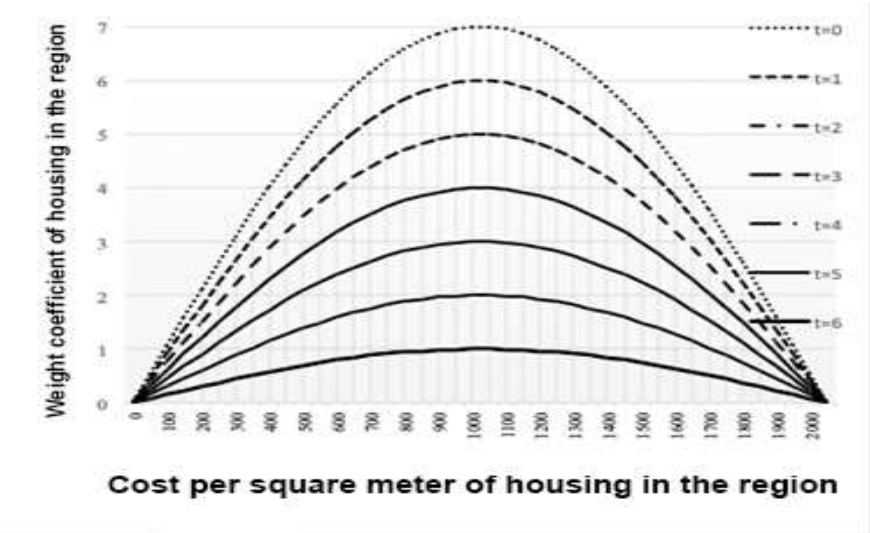

Figure 3 – Evolution of the weight distribution function of the components of the indicator «Average market price per square meter of housing» in the region

Calculations of the average price of a square meter of housing, taking into account the identified weighting distribution function (15) and the criterion of a weighted average assessment (6), and more precisely

$$u_{\text{avr}} = \frac{\sum_{k=1}^{6} w(u_k, 6) \cdot u_k}{\sum_{k=1}^{6} w(u_k, 6)},$$

where $u_1 = u_2 = u_3 = u_4 = u_5 = 800$; $u_6 = 2000$; $w(u_k, 6) = \sin[\pi uk / 2050] \cdot \exp[-(\pi / 2050)2 \cdot 6]$, are summarized in Table 1.

It can be assumed that the obtained average for the indicator «Average market price per square meter of housing» in the form of a weighted average estimate of \$819,21 more objectively reflects the level of housing costs in the region as a whole, rather than the arithmetic average of \$1000.

Table 1 – Assessment of the indicator «Average market price per square meter of housing» on the example of a small region

Code	Regional areas	Amount	Average market price per m^2 of housing ($)	Weight	Average price ($) by formula of	
					arithmetic mean (1)	weighted average estimate (6)
01	Central	1	2000	0,0765	2000	2000
02	Peripheral	5	800	0,9411	4000	4000
Total		6	×	×	1000	819,21

Let us adapt this approach for a region with a much larger number of constituent districts. For this purpose there will be considered a region consisting of 9 districts which differ in their levels of infrastructure development and living conditions. Table 2 shows the average market prices per square meter of real estate in these districts. Assuming an initial condition of the problem (10)–(11) at $t_0 = 0$ and the function $w(u, t_0) = 7\sin[\pi u / 3550]\,\exp[-(\pi / 3550)2 \cdot t_0] = 7\sin[\pi u / 3550]$, we got an automodel solution at the stage t = 6 as the following function (see Fig. 4):

$$w(u, 6) = \sin[\pi u / 3550] \cdot \exp[-6(\pi / 3550)^2].$$

Calculations of the average price for the whole region, taking into account this function and the criterion of weighted average assessment (6), are summarized in Table 2.

Table 2 – Assessment of the indicator «Average market price per square meter of housing» on the example of a medium-sized region

Region	Average market price per m^2 of housing ($)	Weight	Average price ($) by formula of	
			arithmetic mean (1)	weighted average estimate (6)
Central	3500	0,0442	3500	154,82
Region №1	1500	0,9705	1500	1455,79
Region №2	2000	0,9802	2000	1960,47
Region №3	1700	0,9978	1700	1696,25
Region №4	2300	0,8940	2300	2056,19
Region №5	1200	0,8733	1200	1047,96
Region №6	800	0,6503	800	520,23
Region №7	2800	0,6160	2800	1724,93
Region №8	3100	0,3878	3100	1202,14
Sum	18900	6,4142	2100	1842,59

The resulting average for the indicator «Average market price per square meter of housing» in the larger region as a weighted average estimate of $ 1842,59 is slightly different from the arithmetic average of $2100. Nevertheless, the results of the previous example incline towards choosing the weighted average estimate as the most adequate reflection of the «Average market price per square meter of housing» indicator for the region as a whole.

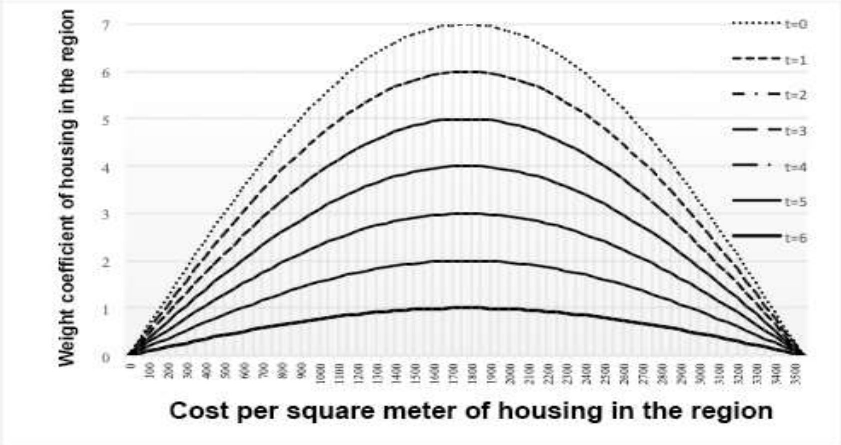

Figure 4 – Evolution of the weight distribution function of the components of the indicator «Average market price per square meter of housing» in the region

The resulting average for the indicator «Average market price per square meter of housing» in the larger region as a weighted average estimate of $ 1842,59 is slightly different from the arithmetic average of $2100. Nevertheless, the results of the previous example incline towards choosing the weighted average estimate as the most adequate reflection of the «Average market price per square meter of housing» indicator for the region as a whole.

6. Conclusions

The averaging method is one of the asymptotic methods for solving the so-called singularly perturbed problems. The most important problems studied by the averaging method arise in the theory of nonlinear oscillations. These fluctuations distinguish the pricing process in the residential real estate market, both in the primary and secondary markets. Within this paradigm, the article discusses a method for calculating the average value of a square meter of housing which is based on the application of a self-similar solution of a partial differential equation with initial and boundary conditions.

REFERENCES

1. Гмурман В.Е. Теория вероятностей и математическая статистика. М.: Высшая школа, 2001. 479 с.
2. Some Motivation. Available on the website: «PennState Statistics Online Courses». URL: https://online.stat.psu.edu/stat414/lesson/24/24.1 (last accessed: 16.03.2021).
3. Mean and Variance of Sample Mean. Available on the website: «PennState Statistics Online Courses». URL: https://online.stat.psu.edu/stat414/lesson/24/24.4 (last accessed: 16.03.2021).
4. Многократные измерения. Available on the website: StudFiles. URL: https://studfile.net/preview/16383383/page:97/ (last accessed: 16.03.2021).
5. How to find the average price of goods and services? Available in the blog: About Trade. URL: https://www.ekam.ru/blogs/pos/srednjaja-cena (last accessed: 16.03.2021).
6. Ganti A. Weighted Average. Available on the website: Investopedia. URL: https://www.investopedia.com/terms/w/weightedaverage.asp: (last accessed: 16.03.2021).
7. Average Price of Goods in Retail. Available on the website: InsoRet. URL: https://insoret.ru/indicators/average_price/ (last accessed: 16.03.2021).
8. Weighted average – formula in Excel. Available on the website: Mister-Office. URL: https://mister-office.ru/formuly-excel/calculate-weighted-average.html (last accessed: 16.03.2021).
9. Rzayev R.R., Aliyev A.R. About One Approach to the Description of Semi-structured Indicators on a Given Data Sample. *Advances in Intelligent Systems and Computing* (Springer's Series). 2020. Vol. 1095. P. 436–444.

10. Курдюмов С.П., Малинецкий Г.Г., Потапов А.Б. Синергетика – новые направления. М.: Знание, 1989. 48 с.
11. Laplace's Equation. Available on the website: Paul's Online Note. URL: https://tutorial.math.lamar.edu/classes/de/laplaceseqn.aspx (last accessed: 16.03.2021).
12. Первая краевая задача для уравнения теплопроводности. Метод Фурье. Построение классического решения. URL: https://www.hse.ru/data/2015/04/23/1095247562/pde15-10-13.pdf (last accessed: 16.03.2021).
13. Heeger D. Linear Systems Theory. URL: http://www.cns.nyu.edu/~david/courses/perception/ lecture-notes/linear-systems/linear-systems.html (last accessed: 16.03.2021).

Стаття надійшла до редакції 15.09.2021