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# ORGANIZATION OF RISK MANAGEMENT IN A COMMERCIAL BANK BASED ON A FUZZY COGNITIVE MAP

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Анотація. Будь-який комерційний банк намагається визначити ступінь допустимих ризиків матеріальних та/або репутаційних втрат, вважаючи, що його можливі втрати обернено пропорційні розміру його капіталу. Отже, основою успішної банківської діяльності є наявність у структурі комерційного банку Системи управління ризиками, основним завданням якої є визначення найкращої (або раціональної) стратегії укладання угод, що забезпечує максимальне зростання прибутку. У рамках системи управління ризиками використовуються інструменти статистичного аналізу, які дозволяють оцінити та порівняти наслідки й доцільність окремих операцій шляхом встановлення кількісної міри банківського ризику від будь-якої операції. Крім того, ці інструменти дозволяють формалізувати різноманітні операції і, таким чином, забезпечити накопичення досвіду комерційних банків. Для формування евристичних знань з історії укладання різноманітних контрактів необхідно використовувати та накопичувати експертні думки і, як наслідок, нечіткі методи аналізу та прийняття рішень щодо банківських операцій по кожній точці локалізації. Розглянуто нечітку когнітивну мапу для оцінки внутрішніх і зовнішніх банківських ризиків. Цей підхід передбачає використання нечіткої когнітивної моделі, на основі якої причинно-наслідкові зв'язки для оцінки банківських ризиків на всіх рівнях ієрархії їх деталізації описуються за допомогою систем нечіткого логічного висновку. Як вхідні та вихідні характеристики використовуються якісні критерії оцінки сфер локалізації банківських операцій та зовнішніх впливів. Для формального опису цих слабоструктурованих характеристик використовуються відповідні нечіткі множини, які відновлюються на відповідних універсумах відповідними функціями належності. При цьому метод точкової оцінки нечітких множин застосовано для дефазифікації нечітких висновків щодо рівнів ризику на всіх вузлах локалізації банківських операцій.

**Ключові слова:** управління ризиками, якісні критерії оцінки, нечітка когнітивна мапа, нечітка множина, система логічного висновку.

Abstract. Any commercial bank tries to determine the degree of acceptable risks of material and/or reputational losses, believing that its potential losses are inversely proportional to the volume of its capital. Therefore, the basis of successful banking is the presence of a Risk Management System in the commercial bank structure, the main task of which is to determine the best (or rational) strategy for concluding contracts and ensuring maximum profit growth. Within the Risk Management System, there are used statistical analysis tools allowing evaluating and comparing the consequences and expediency of certain transactions by establishing a quantitative measure of banking risk from any transaction. Moreover, these tools make it possible to formalize various transactions and, thereby, ensure the accumulation of commercial bank experience. To form heuristic knowledge from the history of the conclusion of various contracts, it is necessary to use and accumulate expert opinions, and, as a result, fuzzy methods of analysis and decisionmaking regarding banking transactions on each localization point. The paper considers a fuzzy cognitive map for assessing internal and external banking risks. This approach involves the use of a fuzzy cognitive model, based on which cause-effect relationships for assessing banking risks at all levels of the hierarchy of their detailing are described using the fuzzy inference systems. Qualitative evaluation criteria for the areas of localization of banking operations and external influences are used as input and output characteristics. For the formal description of these weakly structured characteristics, there are used appropriate fuzzy sets which are restored on the corresponding universes by corresponding membership functions. At the same time, the method of point estimation of fuzzy sets is applied for the defuzzification of fuzzy conclusions relative to the risk levels on all nodes of the localization of banking operations.

*Keywords:* risk management, qualitative evaluation criteria, fuzzy cognitive map, fuzzy set, fuzzy inference system.

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## **1. Introduction**

Any banking activity is carried out under uncertainty, therefore, in the process of implementing its functions and providing financial and credit services, any commercial bank (CB) is inevitably forced to consider many various risks, both commercial and institutional. Since the publication of the book Game Theory and Economic Behavior by J. Neumann and O. Morgenstern, the assessment of banking risks has become one of the main applied areas of modeling risk situations in economics and business. Each CB tries to determine the degree of acceptable risks of financial and/or reputational casualties. At the same time, the potential casualties of the CB are inversely proportional to the volume of its capital. Therefore, in the banking sector of the economy, special attention is paid to the study of the skills of risk management of possible casualties, which allows asserting that the basis of successful banking is the presence of risk management in the structure of the CB. The main task of risk management is to determine the best (or optimal) strategy for making a deal, which ensures the maximum growth of the CB's profit due to the correct multicriteria selection from all potential deals that are distinguished by high rates of profitability and reliability. In other words, risk management is designed to implement the optimal strategy for the allocation of free banking resources by determining the selected set of transactions, which can provide for CB to obtain the maximum average profit with the minimum risk.

To solve this problem relative to all banking operations, within the framework of risk management, the tools of the theory of statistical decisions are used, which, by establishing a quantitative measure of banking risk from a transaction, in each specific case allows evaluating and comparing the consequences and feasibility of certain transactions. Moreover, statistical analysis tools allow the formalization of various transactions and, thereby, ensure the accumulation of CB experience. At the same time, for the formation of heuristic knowledge from the history of the conclusion of various kinds of transactions, it is necessary to use and accumulate expert opinions, and, therefore, fuzzy methods of analysis and decision-making relative to banking transactions.

The use of the apparatus of fuzzy logic in banking becomes possible since, along with quantitative measures of the reliability of banking transactions, it is increasingly necessary to apply their qualitative characteristics. As such an apparatus, it is proposed to use the fuzzy inference mechanism [1], which can combine and aggregate the statistical processing of the results of completed contracts with expert opinions relative to various conditions of their conclusion.

# 2. Problem statement

The main functions of risk management are anticipation, prevention, localization, and elimination of banking solutions with high risk. At the same time, the definition and assessment of banking risks are always relative, and the desire to assign them a numerical value is not always acceptable from the point of view of the further interpretation of complex results. The acceptable level of risk that CB can consider acceptable for itself is a complex concept and cannot be considered as a simple set of its interrelated and/or interdependent components, since each of them is critical. Therefore, when assessing the aggregate banking risk, the numerical averaging of the results for all types of banking operations is not always acceptable. Distinctive features in the process of assessing the aggregate banking risk are the following: 1) incompleteness and uncertainty of the initial information on the composition and nature of factors affecting the magnitude of the risk; 2) the presence of multi-criteria problems in choosing alternatives associated with the need to consider many qualitative factors that determine the level of risk; 3) the impossibility of using classi-

cal optimization methods. Therefore, based on these considerations, it is necessary to develop an adequate model for the integral assessment of the aggregate banking risk.

*The main goal* of the paper is to develop a methodology for the formation of a risk management system for a commercial bank based on the use of a fuzzy cognitive model covering all areas of localization of banking operations.

## 3. Fuzzy cognitive map as the basis for assessing integral banking risk

According to [2], risk is the probability of unfavorable consequences or events, a situation that has the uncertainty of its outcome, or the probability of a possible unwanted loss of something under an unfavorable concurrence of certain circumstances. In the banking sector, risks, as the probability of manifestations of undesirable (sometimes dangerous) factors, manifest themselves not in isolation but in aggregate. As a rule, one risk manifests itself in the composition of another one or is its consequence or cause. Therefore, from the point of view of risk management strategy, the optimal hierarchy should demonstrate the relationships and interdependences between individual groups and types of banking risks.

In the banking sector of the economy, risk is considered as a category of entrepreneurial and institutional activity of CB. It reflects the hidden cause-effect relationship between factors and outcomes. Therefore, the differentiation criteria and features of classification of banking risks that can be identified should be based on the reasons for their occurrence, in the entrepreneurial case, and, in the institutional case, on the differentiation of objects at risk, based on which the consequences of risk realization can be directly observed.

Based on these considerations, a concept, and mechanisms for managing banking risks are being formed that meet the requirements of risk management, which imply the need to consider the essential features of individual risks, which make it possible to form ways to influence them. Therefore, to streamline many banking risks, considering the interdependence spheres of their occurrence and influence, the nature of influence, factors of influence, and by areas of localization, a logical detailing of banking risks is proposed based on a fuzzy cognitive map (FCM) [3], shown in Fig. 1.



Figure 1 – FCM for assessing the aggregate banking risk

In practice, the "interaction" of even two elements of the FCM occurs according to more complex functional laws which are very difficult to formalize in the traditional mathematical form. Therefore, it becomes necessary to apply the mechanism of fuzzy inference to describe the cause-effect relationship between the terms of the aggregate banking risk and to carry out the analysis based on the so-called fuzzy cognitive model (FCMd) [3]. In this case, the nodular factors (concepts) of the FCM are interpreted as fuzzy sets, and the cause-effect relationship between them are established based on a bounded set of fuzzy linguistic rules, which are formed as follows:

"If 
$$x_{k1}$$
 is  $A_{k1}$  and  $x_{k2}$  is  $A_{k2}$  and .... and  $x_{kn}$  is  $A_{kn}$ , then y is  $B_k$ ", (1)

where  $x_{kj}$  ( $j = 1 \div n$ ; k = 1, 2, ...)  $x_{kj}$  are input linguistic variables, characterizing the factors of influence; y is the output linguistic variable that characterizes the level of consolidated risk;  $A_{kj}$  and  $B_k$  are terms (values) of the corresponding input and output linguistic variables which can be described by appropriate fuzzy sets.

#### 4. Description of cause-effect relationships by fuzzy inference systems

In [4], the so-called risk matrix is considered, which the author proposes to use as the information base for assessing and managing banking risks. The basis of this matrix is a scale of the probability of risk occurrence, which is characterized by the following terms:

- A ALMOST CERTAINLY, i.e., a risk situation expected under all circumstances;
- B VERY PROBABLY, i.e., a risk situation is possible almost always;
- C POSSIBLY, i.e., a risk situation occurs from time to time;
- D UNLIKELY, i.e., a risk situation can sometimes happen;
- E OCCASIONALLY, i.e., a risk situation can occur under exceptional circumstances.

The given terms are qualitative criteria for assessing risk situations, which can be described by appropriate fuzzy sets. For this purpose, choosing a discrete set  $U = \{0; 0,25; 0,5; 0,75, 1\}$  as a universe, the corresponding fuzzy sets can be described as:

$$A = \{0/0; 0/0,25; 0/0,5; 0,5/0,75; 1/1\};$$
  

$$B = \{0/0; 0/0,25; 0,5/0,5; 1/0,75; 0,5/1\};$$
  

$$C = \{0/0; 0,5/0,25; 1/0,5; 0,5/0,75; 0/1\};$$
  

$$D = \{0,5/0; 1/0,25; 0,5,5/0,5; 0/0,75; 0/1\};$$
  

$$E = \{1/0; 0,5/0,25; 0/0,5; 0/0,75; 0/1\}.$$

To estimate the level of risk by areas of localization of banking operations, a scale of terms is used, which are described by fuzzy subsets of the discrete universe  $J = \{0; 0,1; 0,2; 0,3; ...; 0,9; 1\}$  with corresponding membership functions  $\mu(j)$  ( $j \in J$ ) in the following form:

• *TL*=TOO LOW: 
$$\mu_{TL}(j) = \begin{cases} 1, j < 1 \\ 0, j = 1 \end{cases};$$

- *VL*=VERY LOW:  $\mu_{VL}(j) = (1-j)^2$ ;
- *ML*=More than low:  $\mu_{ML}(j) = \sqrt{1-j}$ ;
- *L*=LOW:  $\mu_L(j) = 1 j$ ;
- *H*=HIGH:  $\mu_{H}(j) = j$ ;
- *MH*=more than high:  $\mu_{MH}(j) = \sqrt{j}$ ;
- *VH*=VERY HIGH:  $\mu_{VH}(j) = j^2$ ;

• *TH* = TOO HIGH:  $\mu_{TH}(j) = \begin{cases} 1, j = 1, \\ 0, j < 1 \end{cases}$ 

As shown in Fig. 1, the cause-effect relationships can be described using a sufficient set of typical logically consistent rules in the form of (1). Thus, for each of the local concepts of FCM, the following typical fuzzy inference systems (FIS) are chosen.

1. Market risks ( $x_{11}$  – securities risks,  $x_{12}$  – currency risks (exchange operations),  $x_{13}$  – interest rate risks;  $y_1$  – market risks level):

 $e_{11}$ : " $x_{11}$  is E and  $x_{12}$  is E and  $x_{13}$  is E, then  $y_1$  is TL";  $e_{12}$ : " $x_{11}$  is D and  $x_{12}$  is C and  $x_{13}$  is E, then  $y_1$  is VL";  $e_{13}$ : " $x_{11}$  is C and  $x_{12}$  is D and  $x_{13}$  is D, then  $y_1$  is ML";  $e_{14}$ : " $x_{11}$  is D and  $x_{12}$  is C and  $x_{13}$  is C, then  $y_1$  is L";  $e_{15}$ : " $x_{11}$  is C and  $x_{12}$  is C and  $x_{13}$  is B, then  $y_1$  is H";  $e_{16}$ : " $x_{11}$  is B and  $x_{12}$  is B and  $x_{13}$  is B, then  $y_1$  is MH";  $e_{17}$ : " $x_{11}$  is C and  $x_{12}$  is A and  $x_{13}$  is B, then  $y_1$  is VH";  $e_{18}$ : " $x_{11}$  is A and  $x_{12}$  is A and  $x_{13}$  is A, then  $y_1$  is TH".

2. Structural risks ( $x_{21}$  – market risks,  $x_{22}$  – liquidity risks;  $y_2$  – structural risks level):

 $e_{21}$ : " $x_{21}$  is *E* and  $x_{22}$  is *E*, then  $y_2$  is *TL*";  $e_{22}$ : " $x_{21}$  is *D* and  $x_{22}$  is *D*, then  $y_2$  is *VL*";  $e_{23}$ : " $x_{21}$  is *E* and  $x_{22}$  is *C*, then  $y_2$  is *ML*";  $e_{24}$ : " $x_{21}$  is *D* and  $x_{22}$  is *C*, then  $y_2$  is *L*";  $e_{25}$ : " $x_{21}$  is *C* and  $x_{22}$  is *B*, then  $y_2$  is *H*";  $e_{26}$ : " $x_{21}$  is *C* and  $x_{22}$  is *A*, then  $y_2$  is *MH*";  $e_{27}$ : " $x_{21}$  is *B* and  $x_{22}$  is *B*, then  $y_2$  is *VH*";  $e_{28}$ : " $x_{21}$  is *A* and  $x_{22}$  is *A*, then  $y_2$  is *TH*".

3. Counterparty risks ( $x_{31}$  – credit risks,  $x_{32}$  – deposit risks;  $y_3$  – counterparty risks level):

- $e_{31}$ : " $x_{31}$  is *E* and  $x_{32}$  is *E*, then  $y_3$  is *TL*";  $e_{32}$ : " $x_{31}$  is *D* and  $x_{32}$  is *D*, then  $y_3$  is *VL*";  $e_{33}$ : " $x_{31}$  is *E* and  $x_{32}$  is *C*, then  $y_3$  is *ML*";  $e_{34}$ : " $x_{31}$  is *D* and  $x_{32}$  is *C*, then  $y_3$  is *L*";  $e_{35}$ : " $x_{31}$  is *C* and  $x_{32}$  is *B*, then  $y_3$  is *H*";  $e_{36}$ : " $x_{31}$  is *C* and  $x_{32}$  is *A*, then  $y_3$  is *MH*";  $e_{37}$ : " $x_{31}$  is *B* and  $x_{32}$  is *B*, then  $y_3$  is *VH*";
- *e*<sub>38</sub>: "*x*<sub>31</sub> is *A* and *x*<sub>32</sub> is *A*, then *y*<sub>3</sub> is *TH*".

4. Portfolio risks (*x*<sub>41</sub> – counterparty risks, *x*<sub>42</sub> – market risks; *y*<sub>4</sub> – portfolio risks level):

- $e_{41}$ : " $x_{41}$  is E and  $x_{42}$  is E, then  $y_4$  is TL";
- $e_{42}$ : " $x_{41}$  is D and  $x_{42}$  is D, then  $y_4$  is VL";
- *e*<sub>43</sub>: "*x*<sub>41</sub> is *E* and *x*<sub>42</sub> is *C*, then *y*<sub>4</sub> is *ML*";
- *e*<sub>44</sub>: "*x*<sub>41</sub> is *D* and *x*<sub>42</sub> is *C*, then *y*<sub>4</sub> is *L*";
- $e_{45}$ : " $x_{41}$  is C and  $x_{42}$  is B, then  $y_4$  is H";
- $e_{46}$ : " $x_{41}$  is C and  $x_{42}$  is A, then  $y_4$  is MH";
- $e_{47}$ : " $x_{41}$  is B and  $x_{42}$  is B, then  $y_4$  is VH";
- *e*<sub>48</sub>: "*x*<sub>41</sub> is *A* and *x*<sub>42</sub> is *A*, then *y*<sub>4</sub> is *TH*".

5. Financial risks ( $x_{51}$  – portfolio risks,  $x_{52}$  – structural risks,  $x_{53}$  – insolvency risks;  $y_5$  – financial risks level):

 $e_{51}$ : " $x_{51}$  is *E* and  $x_{52}$  is *E* and  $x_{53}$  is *E*, then  $y_5$  is *TL*";  $e_{52}$ : " $x_{51}$  is *D* and  $x_{52}$  is *C* and  $x_{53}$  is *E*, then  $y_5$  is *VL*";  $e_{53}$ : " $x_{51}$  is *C* and  $x_{52}$  is *D* and  $x_{53}$  is *D*, then  $y_5$  is *ML*";  $e_{54}$ : " $x_{51}$  is *D* and  $x_{52}$  is *C* and  $x_{53}$  is *C*, then  $y_5$  is *L*";  $e_{55}$ : " $x_{51}$  is *C* and  $x_{52}$  is *C* and  $x_{53}$  is *B*, then  $y_5$  is *H*";  $e_{56}$ : " $x_{51}$  is *B* and  $x_{52}$  is *B* and  $x_{53}$  is *C*, then  $y_5$  is *MH*";  $e_{57}$ : " $x_{51}$  is *C* and  $x_{52}$  is *A* and  $x_{53}$  is *B*, then  $y_5$  is *VH*";  $e_{58}$ : " $x_{51}$  is *A* and  $x_{52}$  is *A* and  $x_{53}$  is *A*, then  $y_5$  is *TH*".

6. Functional risks ( $x_{61}$  – operational risks,  $x_{62}$  – management risks;  $y_6$  – functional risks level):

 $e_{61}$ : " $x_{61}$  is *E* and  $x_{62}$  is *E*, then  $y_6$  is *TL*";  $e_{62}$ : " $x_{61}$  is *D* and  $x_{62}$  is *D*, then  $y_6$  is *VL*";  $e_{63}$ : " $x_{61}$  is *E* and  $x_{62}$  is *C*, then  $y_6$  is *ML*";  $e_{64}$ : " $x_{61}$  is *D* and  $x_{62}$  is *C*, then  $y_6$  is *L*";  $e_{65}$ : " $x_{61}$  is *C* and  $x_{62}$  is *B*, then  $y_6$  is *H*";  $e_{66}$ : " $x_{61}$  is *C* and  $x_{62}$  is *A*, then  $y_6$  is *MH*";  $e_{67}$ : " $x_{61}$  is *B* and  $x_{62}$  is *B*, then  $y_6$  is *VH*";  $e_{68}$ : " $x_{61}$  is *A* and  $x_{62}$  is *A*, then  $y_6$  is *TH*".

7. Internal risks ( $x_{71}$  – financial risks,  $x_{72}$  – functional risks;  $y_7$  – internal risks level):

 $e_{71}$ : " $x_{71}$  is *E* and  $x_{72}$  is *E*, then  $y_7$  is *TL*";  $e_{72}$ : " $x_{71}$  is *D* and  $x_{72}$  is *D*, then  $y_7$  is *VL*";  $e_{73}$ : " $x_{71}$  is *E* and  $x_{72}$  is *C*, then  $y_7$  is *ML*";  $e_{74}$ : " $x_{71}$  is *D* and  $x_{72}$  is *C*, then  $y_7$  is *L*";  $e_{75}$ : " $x_{71}$  is *C* and  $x_{72}$  is *B*, then  $y_7$  is *H*";  $e_{76}$ : " $x_{71}$  is *C* and  $x_{72}$  is *A*, then  $y_7$  is *MH*";  $e_{77}$ : " $x_{71}$  is *B* and  $x_{72}$  is *B*, then  $y_7$  is *VH*";  $e_{78}$ : " $x_{71}$  is *A* and  $x_{72}$  is *A*, then  $y_7$  is *TH*".

8. Country risks ( $x_{81}$  – economic risks,  $x_{82}$  – political risks,  $x_{83}$  – legal risks;  $y_8$  – country risks level):

 $e_{81}$ : " $x_{81}$  is E and  $x_{82}$  is E and  $x_{83}$  is E, then  $y_8$  is TL";  $e_{82}$ : " $x_{81}$  is D and  $x_{82}$  is C and  $x_{83}$  is E, then  $y_8$  is VL";  $e_{83}$ : " $x_{81}$  is C and  $x_{82}$  is D and  $x_{83}$  is D, then  $y_8$  is ML";  $e_{84}$ : " $x_{81}$  is D and  $x_{82}$  is C and  $x_{83}$  is C, then  $y_8$  is L";  $e_{85}$ : " $x_{81}$  is C and  $x_{82}$  is C and  $x_{83}$  is B, then  $y_8$  is H";  $e_{86}$ : " $x_{81}$  is B and  $x_{82}$  is B and  $x_{83}$  is C, then  $y_8$  is MH";  $e_{87}$ : " $x_{81}$  is C and  $x_{82}$  is A and  $x_{83}$  is B, then  $y_8$  is VH";  $e_{88}$ : " $x_{81}$  is A and  $x_{82}$  is A and  $x_{83}$  is A, then  $y_8$  is TH".

9. Banking risks<sup>1</sup> ( $x_{91}$  – country risks,  $x_{92}$  – internal risks,  $x_{93}$  – force majeure risks;  $y_9$  – banking risks level):

 $e_{91}$ : " $x_{91}$  is E and  $x_{92}$  is E and  $x_{93}$  is C, then  $y_9$  is TL";  $e_{92}$ : " $x_{91}$  is D and  $x_{92}$  is D and  $x_{93}$  is C, then  $y_9$  is VL";  $e_{99}$ : " $x_{91}$  is E and  $x_{92}$  is C and  $x_{93}$  is C, then  $y_9$  is ML";  $e_{94}$ : " $x_{91}$  is D and  $x_{92}$  is C and  $x_{93}$  is C, then  $y_9$  is L";  $e_{95}$ : " $x_{91}$  is C and  $x_{92}$  is B and  $x_{93}$  is C, then  $y_9$  is H";  $e_{96}$ : " $x_{91}$  is C and  $x_{92}$  is A and  $x_{93}$  is C, then  $y_9$  is MH";  $e_{97}$ : " $x_{91}$  is B and  $x_{92}$  is B and  $x_{93}$  is C, then  $y_9$  is VH";

<sup>&</sup>lt;sup>1</sup>Due to the impossibility of forecasting, in all cases force majeure risks are interpreted as uncertain, i.e., as C.

 $e_{98}$ : " $x_{91}$  is A and  $x_{92}$  is A and  $x_{93}$  is C, then  $y_9$  is TH".

The specified linguistic rules in the composition of FISs are quite trivial and can be easily realized in the notation of the MATLAB package. As a result, various banking risk situations can be simulated.

### 5. Assessing the level of country risk using the FIS

Along with force majeure situations, country risks carry the dangers of political, legal, and socioeconomic character. Therefore, to guarantee protection against such threats, it is necessary to consider the economic and political situation in the aggregate (especially in emerging markets), which, in fact, predetermined the intro-duction of the concept of "country risk". Country risk (CR) is a multi-factor category characterized by a combined system of financial and economic, socio-political, and legal factors that distinguish the market of any country [5]. At present, many world rating agencies, and international institutions, such as Euromoney, Institutional Investor, Mood's Investor Service, the European Bank for Reconstruction and Development, the World Bank, etc., are currently ranking countries according to their CR level. At the same time, existing approaches are conditioned by qualitative and/or quantitative, economic, combined, and structurally qualitative methods for estimating of CR. Well-known auditing firm PricewaterhousCoopers uses a limited set of variables to formulate the ratings of the investment attractiveness of states. These variables are formulated and denoted in the following form:  $x_1$  – the presence of corruption;  $x_2$  – compliance with legislation;  $x_3$  – the level of economic growth;  $x_4$  – state policy on accounting and control;  $x_5$  – state regulation [5].

As a multi-criteria procedure, the estimation of the CR level implies the application of the compositional rule for aggregating the obtained results in each specific case. To assess the CR level eight evaluative concepts (terms) are chosen:  $u_1 - \text{"TOO LOW"}$ ,  $u_2 - \text{"VERY LOW"}$ ,  $u_3 - \text{"MORE THAN LOW"}$ ,  $u_4 - \text{"LOW"}$ ,  $u_5 - \text{"HIGH"}$ ,  $u_6 - \text{"MORE THAN HIGH"}$ ,  $u_7 - \text{"VERY HIGH"}$ ,  $u_8 - \text{"TOO HIGH"}$ . More simply,  $C = \{u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8\}$  is a set of criteria for the classification of CR levels. Then, assuming the factors  $x_i$  ( $i = 1 \div 5$ ) as linguistic variables that take their values in the form of denoted terms, the estimation of CR-levels can be carried out using a sufficient set of consistent implicative rules of the form "If <...>, then <...>", based on which the appropriate scale of gradation of the final estimates of CR levels can be established. In this case, basic judgments are formulated as follows:

 $d_1$ : "If there is no corruption in the country and economic development is observed, then the CR-level is acceptable";

 $d_2$ : "If, in addition to the above requirements, a state policy on accounting and control is carried out, then the CR level is more than acceptable";

 $d_3$ : "If, in addition to the conditions specified in  $d_2$ , there is appropriate legislation and government regulation, then the CR level is low";

 $d_4$ : "If there is no corruption, appropriate legislation and economic development are observed, a state policy on accounting and control is carried out, then the CR level is very acceptable";

 $d_5$ : "If there is appropriate legislation, economic development is observed and a state policy on accounting and control is being implemented, but at the same time there is a manifestation of corruption, then the CR level is still acceptable";

 $d_6$ : "If there is corruption in the country, economic development is not observed and government regulation is not carried out, then the CR level is unacceptable".

In these judgments reflecting internal cause-effect relationships, factors  $x_i$  ( $i = 1 \div 5$ ) are input linguistic variables, and the output is a linguistic variable y, the terms of which reflect the CR levels. Then, for specified terms of all linguistic variables, the corresponding implicative rules are as follows:

 $d_1$ : "If  $x_1$ =BE ABSENT and  $x_3$ =OBSERVED, then y=ACCEPTABLE";

- *d*<sub>2</sub>: "If *x*<sub>1</sub>=BE ABSENT and *x*<sub>3</sub>=OBSERVED and *x*<sub>4</sub>=CARRIED OUT, then *y*=MORE THAN ACCEPTABLE";
- *d*<sub>3</sub>: "If  $x_1$ =BE ABSENT and  $x_2$ =EXIST and  $x_3$ =OBSERVED and  $x_4$ =CARRIED OUT and  $x_5$ =IMPLEMENTED, then y=LOW";
- *d*<sub>4</sub>: "If  $x_1$ =BE ABSENT and  $x_2$ =EXIST and  $x_3$ =OBSERVED and  $x_4$ =CARRIED OUT, then y=VERY AC-CEPTABLE";
- *d*<sub>5</sub>: "If *x*<sub>1</sub>=APPEARS and *x*<sub>2</sub>=EXIST and *x*<sub>3</sub>=OBSERVED and *x*<sub>4</sub>=CARRIED OUT, then *y*=ACCEPTABLE";

*d*<sub>6</sub>: "If  $x_1$ =APPEARS and  $x_3$ =NOT VISIBLE and  $x_5$ =NOT IMPLEMENTED, then y=UNACCEPTABLE".

Let the output linguistic variable y is defined on the discrete set  $J = \{0; 0,1; 0,2; ..., 0,9; 1\}$ . Then  $\forall j \in J$  its terms can be described by fuzzy subsets of J using the corresponding membership functions [5, 6]: S=ACCEPTABLE,  $\mu_s(j) = j$ ; MS=MORE THAN ACCEPTABLE,  $\mu_{VS}(j) = j^{(1/2)}$ 

; 
$$L$$
=LOW,  $\mu_L(j) = \begin{cases} 1, j=1, \\ 0, j<1; \end{cases}$ ;  $VS$ =VERY ACCEPTABLE,  $\mu_{VS}(j) = j^2$ ;  $US$ =UNACCEPTABLE,

 $\mu_{\rm US}(j) = 1 - j \, .$ 

Fuzzification of terms in the left-hand sides of the rules is carried out using the Gaussian membership function

$$\mu(u) = e^{-\frac{(u-u_k)^2}{\sigma_i^2}} (i = 1 \div 5), \qquad (2)$$

which restores appropriate fuzzy subsets of the discrete universe *C*, where  $u_k = (a_{k-1} + a_k)/2$ ,  $a_k = 0,125k$  ( $k = 1 \div 8$ ) (see Fig. 2); the density  $\sigma_i^2$  for the *i*-th factor is selected individually under its criticality.



Figure 2 - Uniform gradation of evaluative features for CR factors

Thus, based on (1), the terms from the left-hand sides of the rules can be described by the following fuzzy sets:

• BE ABSENT (the presence of corruption):  $A = \{0,9070/u_1; 0,6766/u_2; 0,4152/u_3; 0,2096/u_4; 0,0870/u_5; 0,0297/u_6; 0,0084/u_7; 0,0019/u_8\};$ 

• EXIST (compliance of legislation):  $B = \{0,9070/u_1; 0,6766/u_2; 0,4152/u_3; 0,2096/u_4; 0,0870/u_5; 0,0297/u_6; 0,0084/u_7; 0,0019/u_8\};$ 

• OBSERVED (economic development):  $C = \{0,9394/u_1; 0,7788/u_2; 0,5698/u_3; 0,3679/u_4; 0,2096/u_5; 0,1054/u_6; 0,0468/u_7; 0,0183/u_8\};$ 

• CARRIED OUT (state policy on accounting and control):  $D = \{0,9497/u_1; 0,8133/u_2; 0,6282/u_3; 0,4376/u_4; 0,2749/u_5; 0,1557/u_6; 0,0796/u_7; 0,0367/u_8\};$ 

• IMPLEMENTED (state regulation):  $E = \{0,9575/u_1; 0,8406/u_2; 0,6766/u_3; 0,4994/u_4; 0,3379/u_5; 0,2096/u_6; 0,1192/u_7; 0,0622/u_8\}.$ 

Considering the introduced formalisms, the implicative rules in symbolic expression are described as follows:

 $d_{1}: (x_{1}=A) \& (x_{3}=C) \Rightarrow (y=S);$   $d_{2}: (x_{1}=A) \& (x_{3}=C) \& (x_{4}=D) \Rightarrow (y=MS);$   $d_{3}: (x_{1}=A) \& (x_{2}=B) \& (x_{3}=C) \& (x_{4}=D) \& (x_{5}=E) \Rightarrow (y=L);$  $d_{4}: (x_{1}=A) \& (x_{2}=B) \& (x_{3}=C) \& (x_{4}=D) \Rightarrow (y=VS);$   $d_{5}: (x_{1} = \neg A) \& (x_{2} = B) \& (x_{3} = C) \& (x_{4} = D) \Rightarrow (y = S);$  $d_{6}: (x_{1} = \neg A) \& (x_{3} = \neg C) \& (x_{5} = \neg E) \Rightarrow (y = US).$ 

Applying the rule of intersection of fuzzy sets [1], the corresponding membership functions are established for the left sides of these rules:

- *d*<sub>1</sub>:  $\mu_{M1}(u) = \min\{\mu_A(u), \mu_C(u)\}, M_1 = \{0,9070/u_1; 0,6766/u_2; 0,4152/u_3; 0,2096/u_4; 0,0870/u_5; 0,0297/u_6; 0,0084/u_7; 0,0019/u_8\};$
- *d*<sub>2</sub>:  $\mu_{M2}(u) = \min\{\mu_A(u), \mu_C(u), \mu_D(u)\}, M_2 = \{0,9070/u_1; 0,6766/u_2; 0,4152/u_3; 0,2096/u_4; 0,0870/u_5; 0,0297/u_6; 0,0084/u_7; 0,0019/u_8\};$
- *d*<sub>3</sub>:  $\mu_{M3}(u) = \min\{\mu_A(u), \mu_B(u), \mu_C(u), \mu_D(u), \mu_E(u)\}, M_3 = \{0,9070/u_1; 0,6766/u_2; 0,4152/u_3; 0,2096/u_4; 0,0870/u_5; 0,0297/u_6; 0,0084/u_7; 0,0019/u_8\};$
- *d*4:  $\mu_{M4}(u) = \min\{\mu_A(u), \mu_B(u), \mu_C(u), \mu_D(u)\}, M_4 = \{0,9070/u_1; 0,6766/u_2; 0,4152/u_3; 0,2096/u_4; 0,0870/u_5; 0,0297/u_6; 0,0084/u_7; 0,0019/u_8\};$
- *d*<sub>5</sub>:  $\mu_{M5}(u) = \min\{1-\mu_A(u), \mu_B(u), \mu_C(u), \mu_D(u)\}, M_5 = \{0,0930/u_1; 0,3234/u_2; 0,4994/u_3; 0,2910/u_4; 0,1453/u_5; 0,0622/u_6; 0,0228/u_7; 0,0072/u_8\};$
- *d*<sub>6</sub>:  $\mu_{M6}(u) = \min\{1 \mu_A(u), 1 \mu_C(u), 1 \mu_E(u)\}, M_6 = \{0, 0425/u_1; 0, 1594/u_2; 0; 3234/u_3; 0, 5006/u_4; 0, 6621/u_5; 0, 7904/u_6; 0, 8808/u_7; 0, 9378/u_8\}.$

As a result, the rules are presented in a more compact form:

 $d_1: (x=M_1) \Rightarrow (y=S);$   $d_2: (x=M_2) \Rightarrow (y=MS);$   $d_3: (x=M_3) \Rightarrow (y=L);$   $d_4: (x=M_4) \Rightarrow (y=VS);$   $d_5: (x=M_5) \Rightarrow (y=S);$  $d_6: (x=M_6) \Rightarrow (y=US).$ 

As a result of the transformation by the Lukasiewicz implication

$$\mu_{U \times J}(u, j) = \min\{1, 1 - \mu_U(u) + \mu_J(j)\}$$
(3)

for each pair  $(u, j) \in U \times J$ , the corresponding fuzzy relations are obtained in the form of matrixes. The intersection of these matrixes generates the following general functional solution *R* 

		0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1 -
	$u_1$	0,0930	0,0930	0,0930	0,0930	0,0930	0,0930	0,0930	0,0930	0,0930	0,0930	0,9575
	<i>u</i> <sub>2</sub>	0,3234	0,3234	0,3234	0,3234	0,3234	0,3234	0,3234	0,3234	0,3234	0,3234	0,8406
	<i>u</i> <sub>3</sub>	0,5006	0,5848	0,5848	0,5848	0,5848	0,5848	0,5848	0,5848	0,5848	0,5848	0,6766
R =	$u_4$	0,7090	0,7904	0,7904	0,7904	0,7904	0,7904	0,7904	0,7904	0,6994	0,5994	0,4994
	$u_5$	0,8547	0,9130	0,9130	0,9130	0,9130	0,8379	0,7379	0,6379	0,5379	0,4379	0,3379
	$u_6$	0,9378	0,9703	0,9703	0,9096	0,8096	0,7096	0,6096	0,5096	0,4096	0,3096	0,2096
	<i>u</i> <sub>7</sub>	0,9772	0,9916	0,9192	0,8192	0,7192	0,6192	0,5192	0,4192	0,3192	0,2192	0,1192
	$u_8$	0,9928	0,9622	0,8622	0,7622	0,6622	0,5622	0,4622	0,3622	0,2622	0,1622	0,0622

reflecting the cause-effect relationships between the factors  $x_i$   $(i = 1 \div 5)$  and the CR levels.

To determine the CR level, there is applied the compositional rule

$$E_k = G_k \circ R \,, \tag{4}$$

where  $E_k$  is the degree of risk acceptability relative to the k-th CR-level  $(k=1\div8)$ ,  $G_k$  is the mapping of the k-th CR level as the fuzzy subset of the discrete universe J. Then, choosing the compositional rule as

$$\mu_{E_k}(j) = \max_{j \in J} \{ \min[\mu_{G_k}(j), \mu_R(j)] \},$$
(5)

and, if in this case

$$\mu_{G_k}(j) = \begin{cases} 0, \ j \neq j_k; \\ 1, \ j = j_k, \end{cases}$$

then

$$\mu_{E_k}(u) = \mu_R(j_k, u),$$

i.e.,  $E_k$  is fuzzy subset of the universe  $J = \{0; 0,1; 0,2; ...; 0.9; 1\}$  with values of corresponding membership function from the *k*-th row of the matrix *R*.

To classify CR-levels by numerical criteria, the procedure of defuzzification of fuzzy outputs of the applied model is used. For example, for the evaluative concept of risk acceptability  $u_1$ , the fuzzy interpretation of the corresponding CR-level is the following fuzzy subset of the universe *J*:  $E_1$ ={0,0930/0; 0,0930/0,1; 0,0930/0,2; 0,0930/0,3; 0,0930/0,4; 0,0930/0,5; 0,0930/0,6; 0,0930/0,7; 0,0930/0,8; 0,0930/0,9; 0,9575/1}. Establishing the level sets  $E_{1\alpha}$  and calculating the corresponding cardinal number  $M(E_{1\alpha})$  by the formula

$$M(E_{1\alpha}) = \frac{1}{m} \sum_{r=1}^{m} x_r .$$
(6)

As a result, we have:

• for 0< $\alpha$ <0,0930:  $\Delta\alpha$ =0.0930;  $E_{1\alpha}$ ={0; 0,1; 0,2; 0,3; 0,4; 0,5; 0,6; 0,7; 0,8; 0,9; 1}.  $M(E_{1\alpha})$ =0,5;

• for 0,0930< $\alpha$ <0,9575:  $\Delta \alpha$ =0,8645;  $E_{1\alpha}$ ={1},  $M(E_{1\alpha})$ =1,0.

To numerically estimate the fuzzy outputs  $E_k$  ( $k = 1 \div 8$ ), the following formula is applied:

$$F(E_k) = \frac{1}{\alpha_{\max}} \int_{0}^{\alpha_{\max}} M(E_{k\alpha}) d\alpha .$$
(7)

In our case, for  $E_1$  we have:

$$F(E_1) = \frac{1}{0,9575} \int_{0}^{0.9575} M(E_{1\alpha}) d\alpha = (0,5 \cdot 0,0930 + 1,0 \cdot 0,8645) / 0,9575 = 0,9514.$$

Similar actions are established point estimates for others fuzzy outputs: for the evaluated concept of risk acceptability  $u_2-F(E_2)=0,8077$ ;  $u_3-F(E_3)=0,5741$ ;  $u_4-F(E_4)=0,4689$ ;  $u_5-F(E_5)=0,3964$ ;  $u_6-F(E_6)=0,3324$ ;  $u_7-F(E_7)=0,2863$ ;  $u_8-F(E_2)=0,2579$ .

In this case,  $F(E_8)=0,2579$  is the smallest defuzzified output of the applied model of the multicriteria assessment of the CR-level. As the upper bound this value corresponds to the consolidated estimate of the CR level "TOO HIGH OR UNACCEPTABLE". Further, from the point of view of the influence of CR factors, the defuzzified output:

- 0,2863 is the upper bound of the qualitive estimate "VERY HIGH OR SIGNIFICANT";
- 0,3324 is the upper bound of the qualitive estimate "MORE THAN HIGH";
- 0,3964 is the upper bound of the qualitive estimate "HIGH";
- 0,4689 is the upper bound of the qualitive estimate "LOW";
- 0,5741 is the upper bound of the qualitive estimate "MORE THAN LOW";
- 0,8077 is the upper bound of the qualitive estimate "VERY LOW OR INSIGNIFICANT";
- 0,9514 is the upper bound of the qualitive estimate "TOO LOW OR NONE.

Within the framework of the accepted assumptions, based on the criterion

$$E = \frac{F(E_k)}{F_{\text{max}}} \times 100, \qquad (8)$$

where  $F(E_k)$  is the estimate of the k-th CR level,  $F_{max}=F(E_1)=0.9514$  in the measure of the interval [0; 100], so it is possible to construct a reasonable scale for assessing the CR levels. The resulting scale is presented in Table 1.

Interval	CR level	Interval	CR level		
(84,90; 100]	TOO LOW	(34,94; 41,66]	HIGH		
(60,34; 84,90]	VERY LOW	(30,09; 34,94]	MORE THAN HIGH		
(49,29; 60,34]	MORE THAN LOW	(27,11; 30,09]	VERY HIGH		
(41,66; 49,29]	LOW	[0; 27,11]	TOO HIGH		

Table 1 – Gradation of CR Levels using the Fuzzy Inference

Now, let's suppose that the specialized expert community is invited to test 10 alternative countries according to a 5-point system:  $a_k(k=1\div10)$  to assess the degree of influence of financial-economic, socio-political, and state-legal factors in these countries on the level of their CR. As a result of expert testing for each factor, consolidated (averaged) expert assessments were obtained and summarized in Table 2.

Table 2 – Averaged expert estimates of the degrees of infidences										
Alternative	Influence factors									
country	$x_1$	$x_2$	$x_3$	$x_4$	$x_5$					
$a_1$	4,5	4,75	4,5	4,75	4,25					
$a_2$	4,85	4,50	4,55	2,75	3,75					
<i>a</i> <sub>3</sub>	3,75	4,00	3,25	3,85	3,25					
$a_4$	4,25	3,45	2,85	2,75	1,85					
<i>a</i> 5	4,00	2,55	3,00	2,25	1,85					
$a_6$	3,55	2,85	2,00	1,25	0,85					
<i>a</i> <sub>7</sub>	2,25	1,75	1,25	1,85	1,50					
$a_8$	2,25	1,85	1,25	0,75	0,25					
$a_9$	5,00	4,75	4,85	4,85	4,75					
$a_{10}$	3,25	2,85	3,75	4,25	3,50					

Table 2 – Averaged expert estimates of the degrees of influences

To build the FIS relative to the estimation of the CR levels, the rules  $d_1 \div d_6$  were chosen as the basis, with the difference that the rules consider 10 hypothetical states  $a_k(k=1\div10)$  as alternatives. In this case, for terms from their left-hand sides of rules  $d_1 \div d_6$  the fuzzification procedure is applied in a slightly different way, namely: each term is reflected in the form of a fuzzy subset of a finite set of evaluated alternatives (in our case, countries)  $\{a_1, a_2, ..., a_{10}\}$  in the form of

$$A_i = \{ \mu_{Ai}(a_1)/a_1, \, \mu_{Ai}(a_2)/a_2, \, \dots, \, \mu_{Ai}(a_{10})/a_{10} \},\$$

where  $\mu_{A_i}(a_i)$  ( $t = 1 \div 10$ ) is the value of the membership function restoring the fuzzy set  $A_i$ , which determines the attitude of the country to the evaluation criterion  $A_i$ . As a membership function, there is chosen a Gaussian function of the form of (2)

$$\mu_{A_i}(u) = e^{-\frac{[e(a_i)-5]^2}{\sigma_i^2}},$$

where  $e_i(a_i)$  is the consolidated assessment of experts to the country  $a_i(t=1\div10)$  by a five-point scale for compliance with the risk for the *i*-th factor as NONEXISTENT;  $\sigma_i^2$  is the density, which is chosen to be the same for all cases of fuzzification as equal to 4.

One of the values of the linguistic variables reflecting the risk situation relative to the factors  $x_i(i=1\div5)$  is the term "NON-EXISTING RISK" which for each case can be represented as a fuzzy subset  $A_i$  of the discrete universe  $U = \{a_1, a_2, ..., a_{10}\}$  as:

•  $A_1 = \{0,9394/a_1, 0,9944/a_2, 0,6766/a_3, 0,8688/a_4, 0,7788/a_5, 0,5912/a_6, 0,151/a_7, 0,151/a_8, 1/a_9, 0,465/a_{10}\};$ 

•  $A_2 = \{0,9845/a_1, 0,9394/a_2, 0,7788/a_3, 0,5485/a_4, 0,2230/a_5, 0,3149/a_6, 0,0713/a_7, 0,0837/a_8, 0,9845/a_9, 0,3149/a_{10}\};$ 

•  $A_3 = \{0,9394/a_1, 0,9506/a_2, 0,4650/a_3, 0,3149/a_4, 0,3679/a_5, 0,1054/a_6, 0,0297/a_7, 0,0297/a_8, 0,9944/a_9, 0,6766/a_{10}\};$ 

•  $A_4 = \{0,9845/a_1, 0,2821/a_2, 0,7185/a_3, 0,2821/a_4, 0,1510/a_5, 0,0297/a_6, 0,0837/a_7, 0,0109/a_8, 0,9944/a_9, 0,8688/a_{10}\};$ 

•  $A_5 = \{0,8688/a_1, 0,6766/a_2, 0,4650/a_3, 0,0837/a_4, 0,0837/a_5, 0,0135/a_6, 0,0468/a_7, 0,0036/a_8, 0,9845/a_9, 0,5698/a_{10}\}.$ 

Considering these formalisms and the above formal descriptions of terms from the righthand sides of the rules  $d_1 \div d_6$ , the basic model can be written in the following form:

$$d_1: (x_1 = A_1) \& (x_3 = A_3) \Longrightarrow (y = S);$$

 $d_{2}: (x_{1}=A_{1}) \& (x_{3}=A_{3}) \& (x_{4}=A_{4}) \Longrightarrow (y=MS);$  $d_{3}: (x_{1}=A_{1}) \& (x_{2}=A_{2}) \& \dots \& (x_{5}=A_{5}) \Longrightarrow (y=L);$  $d_{4}: (x_{1}=A_{1}) \& (x_{2}=A_{2}) \& (x_{3}=A_{3}) \& (x_{4}=A_{4}) \Longrightarrow (y=VS);$  $d_{5}: (x_{1}=\neg A_{1}) \& (x_{2}=A_{2}) \& (x_{3}=A_{3}) \& (x_{4}=A_{4}) \Longrightarrow (y=S);$  $d_{6}: (x_{1}=\neg A_{1}) \& (x_{3}=\neg A_{3}) \& (x_{5}=\neg A_{5}) \Longrightarrow (y=US).$ 

Further, in the usual manner, the intersections of fuzzy sets from the left sides of the rules are found:

- *d*<sub>1</sub>:  $\mu_{M1}(u) = \min\{\mu_{A1}(u); \mu_{A3}(u)\}; M_1 = \{0,9394/a_1; 0,9506/a_2; 0,4650/a_3; 0,3149/a_4; 0,3679/a_5; 0,1054/a_6; 0,0297/a_7; 0,0297/a_8; 0,9944/a_9; 0,4650/a_{10}\};$
- *d*<sub>2</sub>:  $\mu_{M2}(u) = \min\{\mu_{A1}(u); \mu_{A3}(u); \mu_{A4}(u)\}; M_2 = \{0,9394/a_1; 0,2821/a_2; 0,4650/a_3; 0,2821/a_4; 0,1510/a_5; 0,0297/a_6; 0,0297/a_7; 0,0109/a_8; 0,9944/a_9; 0,4650/a_{10}\};$
- *d*<sub>3</sub>:  $\mu_{M3}(u) = \min \{ \mu_{A1}(u); \mu_{A2}(u); \mu_{A3}(u); \mu_{A4}(u); \mu_{A5}(u) \}; M_3 = \{ 0,8688/a_1; 0,2821/a_2; 0,4650/a_3; 0,0837/a_4; 0,0837/a_5; 0,0135/a_6; 0,0297/a_7; 0,0036/a_8; 0,9845/a_9; 0,3149/a_{10} \};$
- *d*<sub>4</sub>:  $\mu_{M4}(u) = \min \{ \mu_{A1}(u); \mu_{A2}(u); \mu_{A3}(u); \mu_{A4}(u) \}; M_4 = \{ 0,9394/a_1; 0,2821/a_2; 0,4650/a_3; 0,2821/a_4; 0,1510/a_5; 0,0297/a_6; 0,0297/a_7; 0,0109/a_8; 0,9845/a_9; 0,3149/a_{10} \};$
- *d*<sub>5</sub>:  $\mu_{M5}(u) = \min \{1 \mu_{A1}(u); \mu_{A2}(u); \mu_{A3}(u); \mu_{A4}(u)\}; M_5 = \{0,0606/a_1; 0,0056/a_2; 0,3234/a_3; 0,1312/a_4; 0,1510/a_5; 0,0297/a_6; 0,0297/a_7; 0,0109/a_8; 0,0000/a_9; 0,3149/a_{10}\};$
- *d*<sub>6</sub>:  $\mu_{M6}(u) = \min \{1 \mu_{A1}(u); 1 \mu_{A3}(u); 1 \mu_{A5}(u)\}; M_6 = \{0,0606/a_1; 0,0056/a_2; 0,3234/a_3; 0,1312/a_4; 0,2212/a_5; 0,4088/a_6; 0,8490/a_7; 0,8490/a_8; 0,0000/a_9; 0,3234/a_{10}\}.$

Thus, because of the transformation of the rules  $d_1 \div d_6$  by the Lukasiewicz implication in the form of (3), for each pair  $(u, j) \in U \times J$  the corresponding fuzzy relations are obtained in the forms of matrixes. The intersection of these matrixes generates the following common functional solution *R* reflecting the cause-effect relationships between the risk situation relative to the influence  $x_i$  ( $i = 1 \div 5$ ) and the CR level.

		0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
	<i>u</i> <sub>1</sub>	0,0930	0,0930	0,0930	0,0930	0,0930	0,0930	0,0930	0,0930	0,0930	0,0930	0,9575
	<i>u</i> <sub>2</sub>	0,3234	0,3234	0,3234	0,3234	0,3234	0,3234	0,3234	0,3234	0,3234	0,3234	0,8406
	<i>u</i> <sub>3</sub>	0,5006	0,5848	0,5848	0,5848	0,5848	0,5848	0,5848	0,5848	0,5848	0,5848	0,6766
<i>R</i> =	<i>u</i> <sub>4</sub>	0,7090	0,7904	0,7904	0,7904	0,7904	0,7904	0,7904	0,7904	0,6994	0,5994	0,4994
	<i>u</i> <sub>5</sub>	0,8547	0,9130	0,9130	0,9130	0,9130	0,8379	0,7379	0,6379	0,5379	0,4379	0,3379
	<i>u</i> <sub>6</sub>	0,9378	0,9703	0,9703	0,9096	0,8096	0,7096	0,6096	0,5096	0,4096	0,3096	0,2096
	<i>u</i> <sub>7</sub>	0,9772	0,9916	0,9192	0,8192	0,7192	0,6192	0,5192	0,4192	0,3192	0,2192	0,1192
	<i>u</i> <sub>8</sub>	0,9928	0,9622	0,8622	0,7622	0,6622	0,5622	0,4622	0,3622	0,2622	0,1622	0,0622

According to (4) and (5), the *k*-th row of the matrix *R* is a fuzzy conclusion relative to the aggregated CR level for the *k*-th alternative country. To numerically interpret each of these fuzzy conclusions, it is necessary to apply the defuzzification procedure. So, for the fuzzy conclusion relative to the CR level of the first country  $E_1$ ={0,0606/0; 0,0706/0,1; 0,1006/0,2; 0,1312/0,3; 0,1312/0,4; 0,1312/0,5; 0,1312/0,6; 0,1312/0,7; 0,1312/0,8; 0,1312/0,9; 0,9394/1}, respectively, we have:

• for 0<a<0,0606:  $\Delta \alpha$ =0,0606;  $E_{1\alpha}$ ={0; 0,1; 0,2; 0,3; 0,4; 0,5; 0,6; 0,7; 0,8; 0,9;1};  $M(E_{1\alpha})$ =0,5;

• for 0,0606< a<0,0706:  $\Delta\alpha$ =0,01;  $E_{1\alpha}=\{0,1;\,0,2;\,0,3;\,0,4;\,0,5;\,0,6;\,0,7;\,0,8;\,0,9;1\};\,M(E_{1\alpha})$  =0,55;

• for 0,0706< $\alpha$ <0,1006:  $\Delta\alpha$ =0,03;  $E_{1\alpha}$ ={0,2; 0,3; 0,4; 0,5; 0,6; 0,7; 0,8; 0,9;1};  $M(E_{1\alpha})$ =0,60;

• for 0,1006< $\alpha$ <0,1312:  $\Delta \alpha$ =0,0306;  $E_{1\alpha}$ ={0,3; 0,4; 0,5; 0,6; 0,7; 0,8; 0,9;1};  $M(E_{1\alpha})$ =0,65;

• for 0,1312 < $\alpha$ <0,9394:  $\Delta \alpha$ =0,8082;  $E_{1\alpha}$ ={1};  $M(E_{1\alpha})$ =1.

According to (7), the numerical estimate of the fuzzy conclusion  $E_1$  is the following:

$$F(E_1) = \frac{1}{0,9388} \int_{0}^{0.9388} M(E_{8\alpha}) d\alpha = 0.9388.$$

Similar actions are established point estimates of fuzzy conclusions relative to the CR-levels for other countries:  $a_2-F(E_2)=0,7687$ ;  $a_3-F(E_3)=0,6047$ ;  $a_4-F(E_4)=0,5370$ ;  $a_5-F(E_5)=0,5206$ ;  $a_6-F(E_6)=0,4552$ ;  $a_7-F(E_7)=0,3055$ ;  $a_8-F(E_8)=0,3001$ ;  $a_9-F(E_9)=0,9927$ ;  $a_{10}-F(E_{10})=0,5140$ . By simply multiplying these values by 100, the final estimates of the CR-levels are obtained in the measure of the [0; 100].

In [6], on a similar example of alternative countries, the corresponding assessments of the CR levels were obtained using the method of weighted multicriteria estimation and the fuzzy method of maximin convolution of qualitative criteria differentiated by their priority. For comparative analysis, the results of these estimates, as well as those obtained in this article, are summarized in Table 3. As can be seen from Table 3, the orders of final estimates of the CR levels only for alternatives  $a_1$ , a and  $a_9$  are the same. In other cases, there is some difference, which is explained by different approaches to the formation of the grading scale for the final estimates of the CR levels. Nevertheless, the fuzzy inference system-based classification of the final estimates is more confident, since in this case the cause-effect relationships between the influence factors and the CR levels are traced, even though these relations are formulated on the basis of trivial but consistent and sufficiently valid implicative rules.

Country	Weighted e	estimation	Maximin co	onvolution	Fuzzy Inference System		
Country	Estimate	Order	Estimate	Order	Estimate	Order	
$a_1$	91,27	2	0,8688	2	93,88	2	
$a_2$	84,62	3	0,2821	5	76,87	3	
<i>a</i> <sub>3</sub>	73,30	4	0,4650	3	60,47	4	
$a_4$	64,47	6	0,0837	6	53,70	5	
<i>a</i> 5	57,64	7	0,0837	7	52,06	6	
$a_6$	47,13	8	0,0135	9	45,52	8	
<i>a</i> <sub>7</sub>	35,54	9	0,0297	8	30,55	9	
$a_8$	29,06	10	0,0036	10	30,01	10	
<i>a</i> 9	97,04	1	0,9845	1	99,27	1	
$a_{10}$	68,55	5	0,3149	4	51,40	7	

Table 3 – Comparative analysis of the results obtained through the use of three methods

## 6. Conclusion

It is known that there is no universal risk management system because the market conditions and structure of all banks are different. For each commercial bank, a separate program should be developed in accordance with its goals and problems. Large banks with many departments need a more developed and well-thought-out risk management system. But the principles and functions of the risk management system are the same for all institutions. For the risk management system to function smoothly, all structural links of the bank, from managerial to operational, should be involved in it. The functions of each unit should be fixed, and the reasons for conflicts of interest should be minimized.

Many methods and tools have been developed to reduce the probability of bank losses. Their effectiveness depends on the ability to choose the right ones, use and customize them for each specific situation. However, in banking, risks cannot be completely avoided, they can only be minimized. To do this, you need to properly build security protection, select the most appropriate methods for assessing and managing hazards. Therefore, the fuzzy approach to assessing banking risks proposed in the article is able, to a certain extent, to mitigate the operational burden on qualified specialists with high professional intuition and knowledge of financial analysis.

# REFERENCES

1. Zadeh L.A. Outline of a new approach to the analysis of complex systems and decision processes. *Trans., Syst., Man., Cybern.* 1973. Vol. SMC-3. P. 28–44.

2. How to determine the acceptable amount of risk. URL: <u>http://fxgear.ru/kak-opredelit-dopustimyj-razmer-riska/</u> (last accessed: 01.10.2022) (in Russian).

3. Kosko B. Fuzzy cognitive maps. *International Journal of Man-Machine Studies*. 1986. Vol. 1. P. 65–75.

4. Prigodich I.A. Bank risk: essence, estimation, procedure, management. *Economics and Management*. 2012. N 4. P. 85–90 (in Russian).

5. PwC Global Risk podcast series. URL: <u>https://www.pwc.com/gx/en/services/advisory/consult-ing/risk/cosoerm-framework/podcasts.html</u> (last accessed: 01.10.2022).

6. Babayeva S.T., Rzayeva I.R., Babayev T. Weighted estimate of country risk using a fuzzy method of maximun convolution. *Advances in Intelligent Systems and Computing. Springer's Series*. 2019. Vol. 896. P. 559–567.

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