https://orcid.org/0000-0001-7756-0004
https://orcid.org/0000-0001-8361-374X

UDC 620.169.1

O.V. FEDUKHIN*, Ar.A. MUKHA*

DETERMINATION OF THE RESIDUAL LIFE OF PRODUCTS UNDER THE INFLUENCE OF SEVERAL DEGRADATION PROCESSES

*Institute of Mathematical Machines and Systems Problems of the National Academy of Sciences of Ukraine, Kyiv, Ukraine

Анотація. *На даний час відома значна кількість результатів досліджень, присвячених вивченню різноманітних видів деградаційних процесів, що протікають у технічних елементах та системах. У більшості робіт кожен із процесів розглядається окремо, а врахування їх впливу на залишковий ресурс виробу зводиться до вибору і урахування домінуючого процесу – процесу, який найбільш інтенсивно розвивається, що представляє в теорії надійності так звану «слабку ланку». Такий підхід значно спрощує деградаційну картину виробу, що призводить до завищених результатів прогнозу. А це є неприпустимим у системах критичного застосування. У статті запропоновано метод, що дозволяє в доступній інженерній формі проводити обчислення залишкового ресурсу об'єктів, які піддаються одночасному впливу кількох процесів деградації з різною частковою участю. Метод заснований на обчисленні середньої швидкості та коефіцієнта варіації узагальненого процесу деградації. Розрахунки проводяться з використанням імовірнісно-фізичного підходу, в рамках якого лежить імовірнісна модель дифузійно-монотонного розподілу відмов (DM-розподілу). Цей розподіл спеціально формалізовано з урахуванням марківського випадкового процесу дифузійного типу з постійною швидкістю. Дифузійно-монотонний розподіл найточніше вирівнює статистичні дані про відмови механічних об'єктів. В умовах впливу кількох деградаційних процесів вперше використані нормалізація вихідних даних та інформація про пайову участь кожного із складових процесів деградації в узагальненому деградаційному процесі. Метод дозволяє уточнити оцінку залишкового ресурсу виробу порівняно з оцінкою, одержаною з урахуванням лише одного домінуючого деградаційного процесу. Отримана уточнена оцінка залишкового ресурсу дозволяє знизити експлуатаційні витрати за рахунок оптимізації міжремонтних інтервалів та встановити реальний термін експлуатації об'єктів, що досліджуються.*
Ключові слова: *ймовірнісно-фізичний підхід, DM-розподіл, узагальнений процес деградації, частковий вплив, залишковий ресурс.*

Abstract. *At present, there are a significant number of research results devoted to the study of various types of degradation processes occurring in technical elements and systems. In most works, each of the processes is analyzed separately, and taking into account their influence on the residual life of the product is reduced to choosing and taking into account the dominant process – the most intensively developing process, representing a so-called "weak link" in the theory of reliability. This approach greatly simplifies the degradation pattern of the product, which leads to overestimated prediction results that are unacceptable in critical application systems. The article proposes a method that in an accessible engineering form allows for calculating the residual resource of objects that are subject to the simultaneous influence of several degradation processes with different partial participation. The method is based on the calculation of the average rate and the coefficient of variation of the generalized degradation process. The calculations are carried out using a probabilistic-physical approach which includes a probabilistic model of a diffusion-monotonous distribution of failures (DM-distribution). This distribution is specially formalized on the basis of a Markov random process of diffuse type with a constant rate. The diffusion-monotonic distribution most accurately aligns the statistical data on failures of mechanical objects. Under the influence of several degradation processes, the normalization of initial data and information on the share par-*

ticipation of each of the constituent degradation processes in the generalized degradation process was used for the first time. The method makes it possible to refine the estimate of the product's residual life in comparison with the estimate obtained by taking into account only one dominant degradation process. The obtained refined estimate of the residual life allows for reducing operating costs by optimizing the overhaul intervals and establishing the actual life of the objects under study.
Keywords: *probabilistic-physical approach, DM-distribution, generalized degradation process, partial impact, residual resource.*

DOI: 10.34121/1028-9763-2023-3-104-112

1. Introduction

The issue of predicting the residual life of technical systems is devoted to a significant number of scientific articles based on the study of the influence of various degradation processes on the life of the systems under study (electronic, electro-mechanical, and mechanical).

Note. Depending on the degree of detail, the system can be either an element or a combination of them, combined into devices and complexes of devices.

When developing models of resource consumption on the basis of the obtained statistical data, graphs of the dependencies of resource parameters on the operating time are constructed, and various methods are proposed for predicting the development trend of the dominant degradation process that affects the object under study [1]. In a number of works, for example [2], the so-called multifactorial models have been developed, but their disadvantage is the complexity of the application and narrow object orientation, which does not allow them to be extended to all classes of technical systems. In the future, such a multifactorial model is needed, this makes it possible to take into account the negative impact of many system degradation processes that occur simultaneously and lead the system to a failure state.

The aim of the article is to develop a method that will allow calculating the residual resource under the influence of several degradation processes.

2. Formulation of the problem

To take into account the influence of several factors of influence on the system, it is proposed:

– to define the failure criteria;

– to evaluate the influence of each of the factors;

– to determine the initial quantitative indicators for each of the influencing factors;

– to set the limit values of indicators for each of the factors of influence;

– to develop a universal mathematical apparatus that takes into account the specifics of the physics of failures of each class of systems.

To build multifactorial models, as a rule, dependencies between several key parameters or characteristics of influencing factors are used, and their influence is set by setting weighting coefficients.

A number of approaches [3–5], which are described in detail in [6], are based on allowable stresses and represent safety margin calculations (for mechanical systems) and are performed in order to comply with the so-called safety criterion. The calculations are based on the calculations of formulas for estimating fracture stresses, which take into account several factors of negative impact on the system, but they do not take into account accidental disease in the processes of the drive mechanisms that cause failure.

In those works where probabilistic [7–9] and semi-probabilistic methods [10] are involved the authors also focus on one prevailing negative factor of influence, then, taking into account the entire range of influences, it is proposed to make several calculations and choose the most pessimistic forecast.

Taking into consideration the shortcomings of single- and multi-factor models for resource estimation listed above, the purpose of this article is to develop a probabilistic-physical calculation method that allows taking into account the influence of several independent (or weakly correlated) degradation processes acting simultaneously on the technical system under study.

3. Method for calculating the residual resource of a technical system in a complex degradation process

3.1. Description of the complex process of product degradation

Any technical product or system is subject to the influence m of independent degradation processes that occur during operation, for example, at a constant rate over time a_j.

The parameters of the generalized product degradation process are determined as in [11]. The average speed of the generalized degradation process is

$$a = \frac{|R|}{t} = \left(\sum_{j=1}^{m} a_j^2 \right)^{1/2},$$

where $|R|$ is the module of the vector function of the determining parameter of the generalized degradation process and is calculated as follows:

$$|R| = \left(\sum_{j=1}^{m} a_j^2 t^2 \right)^{1/2} = t \left(\sum_{j=1}^{m} a_j^2 \right)^{1/2}.$$

Thus, the average speed of the generalized degradation process is calculated according to the formula:

$$a = \left(\sum_{j=1}^{m} a_j^2 \right)^{1/2}. \quad (1)$$

The composite degradation processes are random, and the generalized degradation process can be represented by a monotonic diffusion distribution (DM-distribution [12]) with the shape parameter ν, a consistent estimate of which is the coefficient of variation of the generalized degradation process.

It is possible to obtain an estimate of the shape parameter based on the independence of the components of the degradation processes with the parameters ν_j:

$$\nu = \frac{\left(\sum_{j=1}^{m} \nu_j^2 a_j^2 \right)^{1/2}}{a}. \quad (2)$$

Substituting into (2) the expression for the rate of the generalized degradation process (1), we obtain an estimate for the shape parameter of the diffusion distribution ν:

$$\nu = \frac{\left(\sum_{j=1}^{m} \nu_j^2 a_j^2 \right)^{1/2}}{\left(\sum_{j=1}^{m} a_j^2 \right)^{1/2}}. \quad (3)$$

To build a formalized model of the generalized degradation process of a product, it is necessary to know how the degradation rates for each j-th process are related to the average speed of the generalized degradation process. Assuming that the product degradation process is homogeneous and its components are independent of each other, the expression for the average rate of the generalized degradation process in the application (operation) mode can be written as follows:

$$a_0 = \left(\sum_{j=1}^{m} a_{0j}^2 \right)^{1/2}, \tag{4}$$

where a_{0j} is the rate of the j-th degradation process in the mode of application at a temperature t_0, ^{0}C; a_0 is the rate of the generalized degradation process in the mode of application at a temperature t_0,^{0}C.

Lemma 1. If the generalized degradation process is homogeneous and its components i and k are independent of each other, then the ratio of the rates of degradation processes is proportional to the ratio of failure rates (p_0) for each of them:

$$\frac{a_{0i}}{a_{0k}} = \frac{p_{0i}}{p_{0k}}. \tag{5}$$

In practice, the values a_{0j} are unknown, and the only information about the rates of degradation processes is their share p_{0j} in the generalized degradation process.

Indeed, taking into account (4), (5), the expression for a_{0j} can be written as follows:

$$a_{0j} = \frac{a_0 p_{0j}}{\left(\sum_{j=1}^{m} p_{0j}^2 \right)^{1/2}}. \tag{6}$$

Adding (6) to (3), we get the following:

$$\nu_0 = \frac{1}{a_0} \left(\sum_{j=1}^{m} \frac{\nu_{0j}^2 a_{0j}^2 p_{0j}^2}{\sum_{j=1}^{m} p_{0j}^2} \right)^{1/2} = \left(\sum_{j=1}^{m} \frac{\nu_{0j}^2 p_{0j}^2}{\sum_{j=1}^{m} p_{0j}^2} \right)^{1/2}. \tag{7}$$

Expressions (4), (7) make it possible to estimate the parameters of the diffusion distribution (DN or DM), which describes the generalized process of product degradation for normal use (index 0) when estimating its residual life.

Note. In the future, the values according to formulas (4) and (7) will be obtained for any temperature regime (index 1) with known values of the activation energies of the composite degradation processes E_{aj}.

3.2. Estimation of residual life in case of complex degradation process

If the defining parameter of one product degradation process changes monotonically at a constant average rate, then based on the accepted reliability model, the average residual life is calculated by the formula

$$\tilde{\pi} = \frac{(\Pi_{\text{пр}} - \Pi_1)}{a} \left(1 + \frac{\nu^2}{2} \right), \tag{8}$$

where $\Pi_{\lim}$ is the maximum allowable value of the defining parameter; Π_1 is the measured value of the determining parameter at the time of operation t_n; and ν is the coefficient of variation of the degradation process.

Consider a complex degradation process, for example, for a pipeline, including two degradation processes occurring simultaneously.

1. Erosive-corrosive wear.

Example 1. It is necessary to calculate the residual life of a straight pipeline section after 23 years of operation. 60 similar sites of one year of production and commissioning were subjected to the study [2]. The measurements were carried out for pipes with diameter *D*=530 mm and wall thickness $\mathrm{S_n}$=28 mm. For the period of operation t_n=23 years, the measured wall thickness of the study area was $\mathrm{S_1}$=24 mm at the minimum allowable standard value S_d=19.5 mm.

Calculating the values of the residual resource in absolute units of the initial data

$$\Pi_{\lim} = S_n - \mathrm{S}_d = 28–19.5=8.5 \text{ mm},$$

$$\Pi_1 = S_n - \mathrm{S}_1 = 28–24=4 \text{ mm},$$

$$\nu_1 = 0.35 \text{ (the average value from Table 1)},$$

$$a_1 = \frac{\Delta\varphi_1}{\Delta t_0 n} = \frac{4}{23\cdot 1} = 0.174 \text{ mm/year},$$

$$\tilde{\pi}_1 = \frac{(\Pi_{\lim} - \Pi_1)}{a_1}\left(1+\frac{\nu_1^2}{2}\right) = \frac{8{,}5-4}{0{,}174}\cdot\left(1+\frac{0{,}35^2}{2}\right) = 27.4 \text{ years.}$$

Table 1 – The values of the coefficients of variation for the degradation processes that occur in the system

Type of destruction (degradation process)	Coefficient of variation destruction process
Wear and tear: mechanochemical	0.20–0.50

Let's calculate the values of the final resource in relative units of the initial data. We normalize the data relative to the pipe wall thickness $\mathrm{S_n}$=28 mm.

$$X_{\lim} - \mathrm{X}_1 = \frac{\Pi_{\lim}}{\mathrm{S}_n} - \frac{\Pi_1}{\mathrm{S}_n} = \frac{\Pi_{\lim}\cdot \mathrm{S}_n - \Pi_1\cdot \mathrm{S}_n}{\mathrm{S}_n} = \frac{238-112}{28} = 4{,}5,$$

$$Z = 1 + \frac{\nu_1^2}{2} = 1 + \frac{0{,}35^2}{2} = 1{,}06,$$

$$a_1 = \frac{\Pi_1 - \Pi_0}{\mathrm{t}_n} = \frac{\Pi_1\cdot \mathrm{S}_n - \Pi_0 \mathrm{S}_n}{\mathrm{S}_n \cdot \mathrm{t}_n}, \text{ if } \Pi_0 = 0, \text{ then } a_1 = \frac{\Pi_1}{\mathrm{t}_n} = \frac{4}{23} = 0{,}174 \text{ mm/year},$$

$$\mathrm{Y} = \frac{X_{\lim} - \mathrm{X}_1}{a_1} = \frac{4{,}5}{0{,}174} = 25{,}86,$$

$$\tilde{\pi}_1 = Y \cdot Z = 25{,}86 \cdot 1{,}06 = 27{,}4 \text{ years.}$$

Both approaches in absolute and relative units of data gave identical results in estimating the final resource.

2. Fatigue wear (crack formation).

Example 2. In the example, we calculate the residual life for a pipeline section, its straight section, with an operating time of 23 years of operation. 60 similar sites of one year of production and commissioning were subjected to the study. The indicators are relevant for pipes of straight sections with diameter D = 530 mm and wall thickness $\mathrm{S_n}$ =28 mm. For the period of operation t_n = 23 the measured crack opening width in the wall of the studied area was R_1 = 2 mm at the maximum permissible value $R_{\lim}$ = 4 mm.

Calculating the values of the residual resource in absolute units of the initial data

$$\Pi_{\lim} = R_{\lim} = 4 \text{ mm},$$

$$\Pi_1 = R_1 = 2 \text{ mm},$$

$$\nu_2 = 0.51 \text{ (the average value from Table 2 and Table 3)},$$

$$a_2 = \frac{\Delta\varphi_1}{\Delta t_0 n} = \frac{2}{23\cdot 1} = 0.087 \text{ mm/year},$$

$$\tilde{\pi}_2 = \frac{(\Pi_{\lim} - \Pi_1)}{a_2}\left(1 + \frac{\nu_2^{\,2}}{2}\right) = \frac{2}{0{,}087}\cdot\left(1 + \frac{0{,}51^2}{2}\right) = 26 \text{ years}.$$

Table 2 – The value of the coefficients of variation of working hours and types of destruction processes for typical objects of thermomechanical equipment

Names of objects	The main types of destruction (refusal)	Coefficient variations
Pipelines	Aging, volumetric fatigue of welds and base metal	0.40–0.70

Table 3 – The value of the coefficients of variation for the degradation processes that affect the system

Type of destruction(degradation process)	Coefficient of variation destruction process
Wear and tear:	
mechanochemical	0.20–0.50
Aging	0.40–1.00

Let's calculate the values of the residual resource in relative units of the initial data. We normalize the data with respect to the pipe wall thickness $S_n = 28\, mm$.

$$X_{\lim} - \mathrm{X}_1 = \frac{\Pi_{\lim}}{\mathrm{S_n}} - \frac{\Pi_1}{\mathrm{S_n}} = \frac{\Pi_{\lim}\cdot \mathrm{S_n} - \Pi_1\cdot \mathrm{S_n}}{\mathrm{S_n}} = \frac{4\cdot 28 - 2\cdot 28}{28} = 2{,}0\,,$$

$$Z = 1 + \frac{\nu_2^{\,2}}{2} = 1 + \frac{0{,}51^2}{2} = 1{,}13,$$

$$a_2 = \frac{\Pi_1 - \Pi_0}{\mathrm{t}_n} = \frac{\Pi_1\cdot \mathrm{S}_n - \Pi_0 \mathrm{S}_n}{\mathrm{S}_n\cdot \mathrm{t}_n}\text{, if } \Pi_0 = 0\text{, then } a_2 = \frac{\Pi_1}{\mathrm{t}_n} = \frac{2}{23} = 0{,}087 \text{ mm/year},$$

$$\mathrm{Y}=\frac{X_{\lim}-\mathrm{X}_1}{a_2}=\frac{2}{0,087}=23,$$

$$\tilde{\pi}_2=Y\cdot Z=23\cdot 1{,}13=26\,\text{years}.$$

Both approaches in absolute and relative units also gave identical results for residual resource estimation.

Estimation of the residual resource under the conditions of a complex degradation process

Option 1. For each of the degradation processes, we calculate $\tilde{\pi}_1$ and $\tilde{\pi}_2$, and the resulting residual resource can be represented as [13]:

$$\tilde{\pi}_{3.1}=\frac{1}{\left(\frac{1}{\tilde{\pi}^2{}_1}+\frac{1}{\tilde{\pi}^2{}_2}\right)^{\frac{1}{2}}},$$

where $\tilde{\pi}_1$ =27.4 years, $\tilde{\pi}_2$ =26 years.

$$\tilde{\pi}_{3.1}=\frac{1}{\left(\frac{1}{\tilde{\pi}^2{}_1}+\frac{1}{\tilde{\pi}^2{}_2}\right)^{\frac{1}{2}}}=\frac{1}{\left(\frac{1}{27{,}4^2}+\frac{1}{26^2}\right)^{\frac{1}{2}}}=18.9\text{ years.}$$

Option 2. Let's assume that, based on the results of the operation of products (for example, pipelines), it is known that the share of failures caused by the first degradation process (erosive-corrosive wear) accounts for p_1 share of failures, and the share of the second degradation process (fatigue wear) accounts for p_2 share of failure rate, for example, p_1=0.8 and p_2=0.2. So $p_1+p_2=1$.

For further calculations, we use the values $\alpha_1=0{,}174$ and $\alpha_2=0{,}087$, obtained from the normalized initial data in relative units. Let's calculate the value of the coefficient of variation of the generalized degradation process according to (7) for $\nu_1=0{,}35$ and $\nu_2=0{,}51$:

$$\nu=\left(\sum_{j=1}^{m}\frac{\nu_j^2 p_j^2}{\sum_{j=1}^{m}p_j^2}\right)^{1/2}=\left(\frac{0{,}35^2\cdot 0{,}8^2}{0{,}8^2+0{,}2^2}+\frac{0{,}51^2\cdot 0{,}2^2}{0{,}8^2+0{,}2^2}\right)^{1/2}=0.36.$$

Let's calculate the value of the average degradation rate:

$$a=\left(\sum_{j=1}^{m}a_j^2\right)^{1/2}=\left(0{,}174^2+0{,}087^2\right)^{1/2}=0.194\text{ mm/year},$$

$$Z=1+\frac{\nu^2}{2}=1+\frac{0{,}36^2}{2}=1{,}065.$$

Let's normalize the following initial data with respect to $S_n=28\,mm$ and calculate the following:

$$X_{\lim 1}=\frac{\Pi_{\lim 1}}{\mathrm{S}_n}=\frac{8{,}5}{28}=0.304,$$

$$X_{\lim 2} = \frac{\Pi_{\lim 2}}{S_n} = \frac{4}{28} = 0.143,$$

$$X_{\lim.avg.} = \left(X_{\lim 1} + X_{\lim 2}\right)/2 = \left(0,304 + 0,143\right)/2 = 0.224,$$

$$X_{1.1} = \frac{\Pi_{1.1}}{S_n} = \frac{4,0}{28} = 0.143,$$

$$X_{1.2} = \frac{\Pi_{1.2}}{S_n} = \frac{2}{28} = 0.071,$$

$$X_{1avg} = \left(X_{1.1} + X_{1.2}\right)/2 = \left(0,143 + 0,071\right)/2 = 0.107,$$

$$X_{\lim} - X_{1s} = \frac{\Pi_{\lim.avg.}}{S_n} - \frac{\Pi_{1avg}}{S_n},$$

$$\left(X_{\lim.avg.} - X_{1avg}\right)\cdot S_n = \Pi_{\lim.avg.} - \Pi_{1avg} = \left(0,224 - 0,107\right)\cdot 28 = 3.276.$$

Calculating the average residual resource is then made as follows:

$$\tilde{\pi}_{3.2} = \frac{(\Pi_{\lim} - \Pi_{1avg})}{a}\cdot Z = \frac{3,276}{0,194}\cdot 1,065 = 17.98 \text{ years.}$$

4. Conclusions

All the methods for estimating the residual life given in the article showed results that are convergent in the order of magnitude. The methods that take into account the simultaneous occurrence of several degradation processes in the product should be considered the most adequate. The paper proposes a method that makes it possible, on the basis of the participation of component degradation processes, to evaluate the parameters of the diffusion distribution and based on them correctly calculate the residual life of the product. Methods with scores $\tilde{\pi}_{3.1}$ and $\tilde{\pi}_{3.2}$ led to the closest results, and the choice of one of them is recommended to be made only on the basis of the availability and completeness of the initial data on the degradation processes occurring in the product.

REFERENCES

1. Гиря М.П. Прогнозирование остаточного ресурса электрооборудования АЭС. *Энергоснабжение. Энергетика. Энергоаудит*. 2010. № 8. С. 25–32.

2. Бараненко В.И., Гулина О.М., Сальников Н.Л., Мурзина О.Э. Обоснование расчетов скорости эрозионно-коррозионного износа и остаточного ресурса трубопроводов АЭС по данным эксплуатационного контроля. *Известия вузов. Ядерная энергетика*. 2016. № 2. С. 5–65.

3. Qu C. Study on Residual Life Prediction METHOD of Corrosive Defective Pipeline: master’s Thesis. Shenyang, China: Northeast University, 2014. P. 18.

4. The American Society of Mechanical Engineers. Manual for Determining the Remaining Strength of Corroded Pipelines. New York, NY, USA: ASME, 1984. P. 1–38.

5. Ma B., Shuai J., Wang J., Kejiang H. Analysis on the latest assessment criteria of ASME B31G-2009 for the remaining strength of corroded pipelines. *Journal of Failure Analysis and Prevention.* 2011. N 11. P. 666–671.

6. Li H., Huang K., Zeng Q., Chong S. Residual Strength Assessment and Residual Life Prediction of Corroded Pipelines: A Decade Review. *Energies*. 2022. N 15 (3). P. 726.

7. Федухін О.В., Муха Ар. А. Оцінка залишкового ресурсу вертикальних циліндричних сталевих резервуарів для нафтопродуктів з урахуванням геометричних конструктивних відхилень. *Математичні машини і системи.* 2022. № 4. С. 100–107.
8. Федухін О.В., Муха Ар.А. Оцінка залишкового ресурсу контактного проводу електрофікованих залізничних доріг. *Математичні машини і системи.* 2022. № 2. С. 91–101.
9. Федухін О.В., Муха Ар.А. Оцінка залишкового ресурсу трубопроводу в умовах ерозійно-корозійного зношування. *Математичні машини і системи.* 2023. № 2. С. 88–93.
10. Noor N.M., Yahaya N., Ozman N.A.N., Othman S.R. The Forecasting Residual Life of Corroding Pipeline based on Semi-Probabilistic Method. *UNIMAS e-Journal of Civil Engineering*. 2010. Vol. 1 (2). P. 246–263.
11. Федухин А.В. Ускоренная оценка надежности электронных элементов и устройств по результатам форсированных испытаний. К.: Інтерсервіс, 2019. 200 с.
12. Стрельников В.П., Федухин А.В. Оценка и прогнозирование надёжности электронных элементов и систем. К.: Логос, 2002. 486 с.
13. ДСТУ 8646:2016. Надійність техніки. Оцінювання та прогнозування залишкового ресурсу (строку служби) технічних систем. [Чинний від 2017-07-01]. Київ, 2017. 69 с.

Стаття надійшла до редакції 10.08.2023