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DETERMINING RESIDUAL PRODUCT LIFE CONSIDERING MULTIPLE DEGRADATION PROCESSES AND VARIABLE LOADING

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Анотація. На сьогоднішній день відзначаються значні досягнення у сфері, що відносяться до аналізу різних типів деградаційних процесів, які протікають у технічних елементах та системах. У більшості досліджень кожен із процесів розглядається окремо, а комплексна оцінка впливу усіх деградаційних процесів на кінцеве значення залишкового ресурсу обмежується урахуванням одного домінуючого процесу, того, що розвивається найінтенсивніше, відомого у теорії надійності як «слабка ланка». Використання цього методу вносить значні спрощення в загальну картину деградації і сприяє надмірності у кінцевому прогнозі, що є неприйнятним у системах критичного застосування. У статті запропоновано підхід, який у доступній інженерній формі дозволяє провести необхідні розрахунки для оцінки залишкового ресурсу виробу, що одночасно знаходиться під впливом кількох деградаційних процесів із різною часткою участі та змінним температурним навантаженням. Цей метод ґрунтується на обчисленні середньої швидкості та коефіцієнта варіації узагальненого процесу деградації. Розрахунки виконуються за допомогою ймовірнісно-фізичного підходу, в рамках якого лежить імовірнісна модель у вигляді дифузійного монотонного розподілу відмов (DM-розподілу). Розподіл представлено з залученням марковського випадкового процесу дифузійного типу з постійною швидкістю, що належним чином адаптує і вирівнює дані по статистиці відмов у механічних об'єктах. Під впливом декількох процесів деградації вперше використано нормалізацію вихідних даних та інформацію про долю кожного з компонентів у загальному процесі деградації. Застосування цього підходу дозволяє більш точно визначити залишковий ресурс виробу у порівнянні з оцінкою, отриманою при урахуванні тільки одного домінуючого процесу деградації. Деталізована оцінка залишкового ресурсу, яку отримано після уточнень, сприяє зниженню експлуатаційних витрат через оптимізацію інтервалів між технічним обслуговуванням та встановленню реального терміну служби об'єктів дослідження.

Ключові слова: ймовірнісно-фізичний підхід, DM-розподіл, узагальнений процес деградації, частковий вплив, залишковий ресурс.

Abstract. As of today, significant progress has been observed in the field related to the analysis of various types of degradation processes occurring in technical elements and systems. In most studies, each of these processes is examined individually, and the comprehensive assessment of the impact of all degradation processes on the final residual life value is limited by considering a single dominant process — the one that develops most intensively, known in reliability theory as «the weakest link». Applying this method simplifies the overall degradation profile significantly and contributes to an overestimation in the final prediction which is unacceptable in critical application systems. The article proposes an approach that, in an accessible engineering form, allows for the necessary calculations to assess the residual life of a product that is simultaneously under the influence of multiple degradation processes with varying degrees of participation and temperature loads. This approach relies on calculating the mean rate and coefficient of variation for the generalized degradation process. The calculations are conducted employing a probabilistic-physical approach, within which lies a probability model in the form of a diffusion monotonic failure distribution (DM-distribution). This distribution is presented with the involvement of a Markov diffusion-type random process with a constant speed which properly adapts and aligns statistical failure data of

mechanical objects. Being influenced by multiple degradation processes, the normalization of the initial data and information regarding the proportions of each of the components in the general degradation process has been used for the first time. The application of this approach allows for a more accurate estimation of the product's residual life compared to the estimation obtained by considering only one dominant degradation process. The detailed assessment of the residual life, which is obtained after refinements, contributes to reducing operational costs through the optimization of technical maintenance intervals and establishing the actual service life of the researched objects.

Keywords: *probabilistic-physical approach, DM-distribution, generalized degradation process, partial influence, residual life.*

1. Problem statement

A significant number of scientific articles are focused on predicting the residual life of technical systems. These articles are based on the study of how various degradation processes influence the lifespan of the systems under investigation (electronic, electromechanical, and mechanical).

Note 1. Depending on the level of detail, a system can be represented as either an individual element or as a combination of elements assembled into devices and device complexes.

When developing resource expenditure models based on acquired statistical data, dependencies between resource parameters and operational time are determined, along with the proposal of various methods for forecasting the dominant degradation process trend that affects the studied object [1]. In some studies, so-called multifactor models have been developed. However, their drawback lies in the complexity of implementation and their limited object-oriented focus, which restricts their applicability to all categories of technical systems. As time progresses, a demand will emerge for a multifactor model that allows considering the impact of several concurrent resource degradation processes, collectively leading the system towards a failure state.

In order to consider the impact of multiple influencing factors on the system, the following is suggested:

– to determine failure criteria;

– to assess the impact of each aspect;

– to define the starting quantitative parameters for each of the influencing variables;

– to establish limit values for each influencing factor;

– to develop a universal mathematical tool that takes into account the specifics of failure physics for each system class.

When constructing multifactor models, it is a common practice to utilize dependencies among several key parameters or characteristics of influencing factors, with their impact specified by establishing weight coefficients.

A series of approaches [2–4], extensively explained in [5], are based on allowable stresses and represent calculations of safety margins (for mechanical systems) to secure conformity with the established safety criterion. The basis for the calculations is the formulas for fracture stress assessment, considering multiple detrimental factors affecting the system but not considering the random nature of the failure physical processes themselves.

In those works where probabilistic and semi-probabilistic methods [6] are involved, authors also emphasize a predominant negative influencing factor. To account for the entire spectrum of influences, it is proposed to perform multiple calculations and select the most pessimistic forecast.

Considering the above-mentioned disadvantages of single and multifactor resource assessment models, *the aim of this article is* to create a probabilistic-physical calculation approach that allows for considering the impact of multiple independent degradation processes concurrently affecting the studied technical system.

2. Main part

2.1. Description of a complex product degradation process

During operation, every technical product or system experiences the impact of m independent degradation processes, occurring at a constant rate a_j.

The determination of parameters for the generalized product degradation process is conducted as follows. The average rate of the generalized degradation process is calculated as

$$a = \frac{|R|}{t} = \left(\sum_{j=1}^{m} a_j^2 \right)^{1/2},$$

where $|R|$ is the vector-function module of the defining parameter for the generalized degradation process which is calculated as follows:

$$|R| = \left(\sum_{j=1}^{m} a_j^2 t^2 \right)^{1/2} = t \left(\sum_{j=1}^{m} a_j^2 \right)^{1/2}.$$

Thus, the average rate of the generalized degradation process is calculated by the formula

$$a = \left(\sum_{j=1}^{m} a_j^2 \right)^{1/2}. \tag{1}$$

The constituent degradation processes are random, while a monotonic diffusion distribution can represent the generalized process of degradation (*DM*-distribution) with scale parameters a and shape v [7]. The credible estimation of v is represented by the coefficient of variation V for the generalized degradation process.

Estimating the shape parameter becomes achievable when taking into account the independence of the degradation process components with parameters v_j:

$$v = \frac{\left(\sum_{j=1}^{m} v_j^2 a_j^2 \right)^{1/2}}{a}. \tag{2}$$

An estimate for the diffusion distribution shape parameter v is obtained by substituting the expression for the rate of the generalized degradation process (1) into equation (2)

$$v = \frac{\left(\sum_{j=1}^{m} v_j^2 a_j^2 \right)^{1/2}}{\left(\sum_{j=1}^{m} a_j^2 \right)^{1/2}}. \tag{3}$$

To formulate a formalized model for the product's generalized degradation process, it is crucial to grasp the connection between the degradation rates for each j-th process and the mean rate of the generalized degradation process. Given that the degradation process of the product is homogeneous and its components are independent, the expression for the mean rate of the generalized degradation process during normal operation can be expressed as follows:

$$a_0 = \left(\sum_{j=1}^{m} a_{0j}^2 \right)^{1/2}, \tag{4}$$

where a_{0j} is the rate of the j-th degradation process under operating conditions at temperature t_0, °C; a_0 is the rate of the generalized degradation process under operating conditions at temperature t_0, °C.

Statement. When the generalized degradation process is homogeneous, and its constituents i and k are not correlated, then the ratio of degradation process rates is directly proportional to the ratio of failure rates (p_0) for each of them:

$$\frac{a_{0i}}{a_{0k}} = \frac{p_{0i}}{p_{0k}}. \tag{5}$$

In practice, the values a_{0j} are often unknown, and the only information available about the degradation process rates is their fractional participation p_{0j} in the generalized degradation process.

Indeed, considering (4) and (5), the expression for a_{0j} can be written as follows:

$$a_{0j} = \frac{a_0 p_{0j}}{\left(\sum\limits_{j=1}^{m} p_{0j}^2 \right)^{1/2}}. \tag{6}$$

By substituting (6) in (3), we get

$$V_0 = v_0 = \frac{1}{a_0} \left(\sum\limits_{j=1}^{m} \frac{v_{0j}^2 a_{0j}^2 p_{0j}^2}{\sum\limits_{j=1}^{m} p_{0j}^2} \right)^{1/2} = \left(\sum\limits_{j=1}^{m} \frac{v_{0j}^2 p_{0j}^2}{\sum\limits_{j=1}^{m} p_{0j}^2} \right)^{1/2}. \tag{7}$$

Expressions (4) and (7) enable parameter estimation for the diffusion distribution (*DN* or *DM*) describing the generalized product degradation process under normal operating conditions (index $_0$) when assessing its residual life.

Note 2. Further on, values can be acquired for any temperature conditions (index $_1$) using the expressions (4) and (7) if the activation energy values of the constituent degradation processes E_{aj} are known.

Let p_{ij} denote the failure rate for the j-th degradation process in accelerated (forced) mode, distinct from the operating condition at temperature T_0. Due to the fact that degradation processes under consideration have different activation energies when transitioning to accelerated mode with higher temperatures, there is a redistribution of failure rates for each process or the so-called transformation of the Pareto chart. Furthermore, the connection between p_{0j} and p_{1j} in this case is described by the expression

$$p_{1j} = \frac{p_{0j} K_{f1j}}{\sum\limits_{j=1}^{m} p_{0j} K_{f1j}}, \tag{8}$$

where K_{f1j} ($_f$ stands for forcing/acceleration) is the acceleration coefficient for the degradation rate of the j-th process when transitioning from t_0 to t_1.

The relationship between the degradation rates of constituent processes in operating and accelerated modes is determined by expressions of the following kind:

$$a_{1j} = a_{0j} K_{f1j}.$$ (9)

For example, for three constituent processes

$$a_{11} = a_{01} K_{f11},$$

$$a_{12} = a_{02} K_{f12},$$

$$a_{13} = a_{03} K_{f13}.$$

For processes that have no thermal component, as well as for random processes unrelated to physical processes (resulting in failures classified as «miscellaneous»), the coefficient K_{f1j} is assumed to be equal to one. For other thermally activated processes, K_{f1j} is calculated using the formula

$$K_{f1j} = \exp\left[\frac{E_{aj}}{K}\left(\frac{1}{T_0} - \frac{1}{T_1}\right)\right],$$ (10)

where K is the Boltzmann constant: $8,6173 \cdot 10^{-5}$ eV (Electronvolt) K^{-1} (Kelvin); E_{aj} is the activation energy, eV; T_0 and T_1 are absolute products temperatures, respectively, in the operating and accelerated modes ($T = t°C + 273$).

For example, it is assumed that there is one dominant degradation process in the product with an activation energy of $E_a = 0.8\,\text{eV}$. The operating temperature is $t_0 = +60\,°C$, and the accelerated mode temperature is $t_1 = +170\,°C$. Calculating the temperature values in absolute units looks as follows:

$$T_0 = t_0 + 273 = 333\,°K\,(+60°C),$$

$$T_1 = t_1 + 273 = 443\,°K\,(+170°C).$$

Determining the expected acceleration factor is carried out in the following way:

$$K_{f11} = \exp\left[\frac{E_a}{K}\left(\frac{1}{T_0} - \frac{1}{T_1}\right)\right] = \exp\left[\frac{0,8}{8.6173 \cdot 10^{-5}}\left(\frac{1}{333} - \frac{1}{443}\right)\right] = 1014.8626.$$

Fig. 1 shows, as an example, a Pareto chart for a product, where at $t_0 = 55°C$, there are four degradation processes with their corresponding characteristics:

$$\{E_{a1} = 0.46;\ V_{01} = 1.1;\ p_{01} = 0.60\},$$

$$\{E_{a2} = 0.70;\ V_{02} = 0.7;\ p_{02} = 0.15\},$$

$$\{E_{a3} = 0.70;\ V_{03} = 0.6;\ p_{03} = 0.10\},$$

$$\{E_{a4} = 0;\quad V_{04} = 1.0;\ p_{04} = 0.15\}.$$
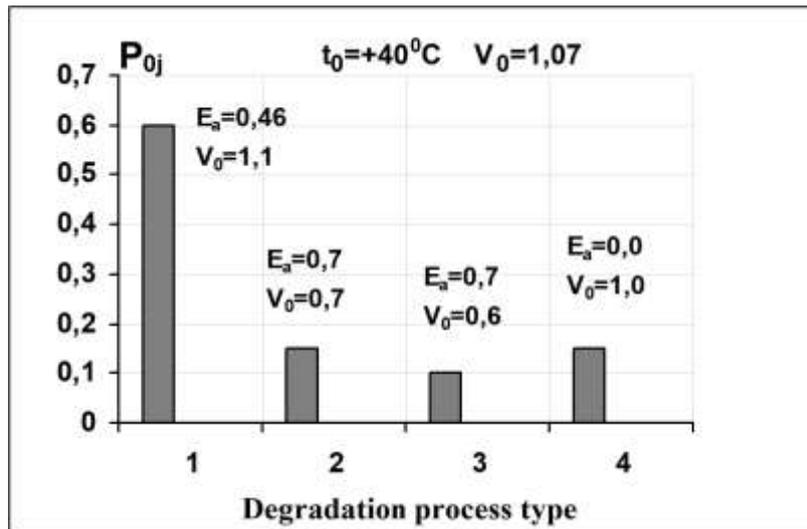
Figure 1 — Initial Pareto chart for $t_0 = +40 \, °C$

Fig. 2 demonstrates the transformed Pareto chart. The transformation was based on (7), (8), (10), and (11) starting from the normal operating temperature $t_0 = +40 \, °C$ to the elevated (accelerated mode) temperature of $t_1 = +125 \, °C$. It becomes apparent that with an increase in temperature to $+125 \, °C$, there is a shift in dominance between the first and second degradation processes. In the operating mode, the process with an activation energy of $E_a = 0.46 \, \text{eV}$ dominated, resulting in 60% of failures. In testing mode, the impact of the first process decreased to 26%, while the impact of the second process increased to 43%.
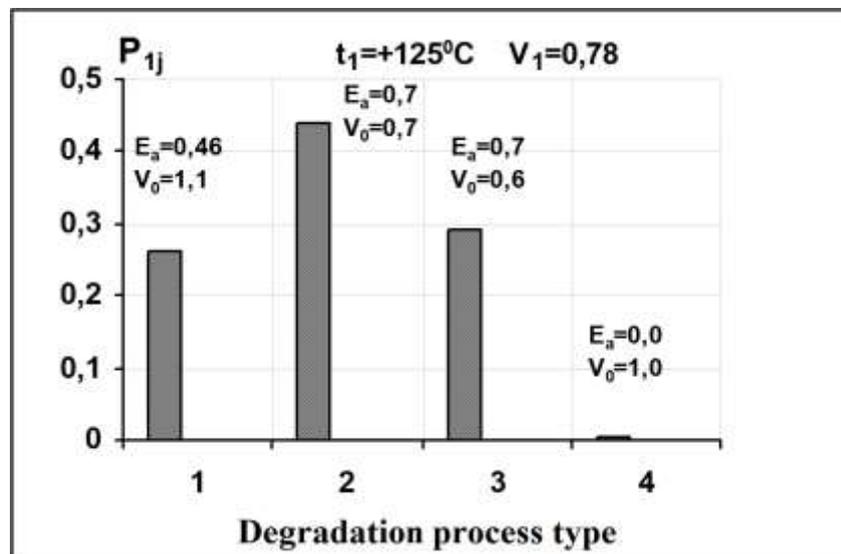
Figure 2 — Transformed Pareto chart at the accelerated temperature mode of $t_1 = +125 \, °C$

Note 3. The variation coefficient of product time-to-failure V is a dimensionless value, and from the probabilistic-physical approach perspective, characterizes the generalized degradation process degree of variability. The coefficient of variation can also indicate the stability and consistency of the constituent degradation processes occurring in the product. A quantitative variability measure, the coefficient of variation, holds significant theoretical and economic importance.

The Pareto chart transformation enables the evaluation of the variation coefficient for the generalized degradation process under any accelerated mode.

In the example mentioned earlier, redistributing the failure proportions among the constituent degradation processes leads to a change in the variation coefficient of the generalized degradation process in the accelerated mode.

For example, assuming the hypothesis of equal variation coefficients for constituent degradation processes in operating and accelerated modes, we get the following:

$$V_1 = v_1 = \left(\sum_{j=1}^{m} \frac{v_{1j}^2 p_{1j}^2}{\sum_{j=1}^{m} p_{1j}^2} \right)^{1/2} = \left(\sum_{j=1}^{m} \frac{v_{0j}^2 p_{1j}^2}{\sum_{j=1}^{m} p_{1j}^2} \right)^{1/2} = 0.78 . \tag{11}$$

Thus, when the thermal load changes, a phenomenon of shifting dominance between degradation processes with different activation energies may occur, reflected in a significant alteration of the fractional contribution values for constituent degradation processes in the generalized degradation process.

To consider this phenomenon, an expression has been derived that enables the calculation of fractional contributions of constituent degradation processes (8) in any temperature-accelerated mode, which forms the basis for the Pareto chart transformation. It is apparent that in the accelerated mode at a temperature of $t_1 = +125\,°C$, a shift in dominance occurs among the constituent degradation processes, towards those with higher activation energies.

2.2. Determining the residual life in case of a complex degradation process

When the defining parameter of a product's degradation process changes monotonically with a constant average speed, then, according to the adopted reliability model, the average residual life is calculated using the formula [7, 8]

$$\tilde{\pi} = \frac{(P_{\lim} - P_1)}{a} \left(1 + \frac{v^2}{2} \right), \tag{12}$$

where $P_{\lim}$ is the maximum allowable value of the determining parameter; P_1 is the measured value of the defining parameter at the operating time t_{ot}; v is the coefficient of variation for the degradation process.

Let's consider a complicated degradation process, like the one in a pipeline that has two degradation processes occurring simultaneously.

Example 1. Erosion-corrosion wear

After 23 years in service, let's estimate the residual life of a straight section of the pipeline. A total of 60 similar sections, all produced and put into operation in the same year, were subjected to the study. Measurements were conducted for pipes with a diameter of $D = 530\,mm$ and a nominal wall thickness of $S_{nom} = 28\,mm$. During the operational period of time $t_{ot} = 23$ years, the measured wall thickness of the examined section was $S_1 = 24\,mm$, with a minimum permissible normative value of $S_p = 19.5\,mm$.

Calculating the residual life value in relative units of the initial data. For this purpose, we will normalize the data relative to the pipe wall thickness $S_{nom} = 28\,mm$:

$$P_{\lim} = S_{nom} - S_p = 28 - 19.5 = 8.5\,mm,$$

$$P_1 = S_{nom} - S_1 = 28 - 24 = 4 \text{ mm},$$

$$X_{\lim} - X_1 = \frac{P_{\lim}}{S_{nom}} - \frac{P_1}{S_{nom}} = \frac{P_{\lim} \cdot S_{nom} - P_1 \cdot S_{nom}}{S_{nom}} = \frac{238 - 112}{28} = 4.5,$$

$$Z = 1 + \frac{v_1^2}{2} = 1 + \frac{0.35^2}{2} = 1.06,$$

$$a_1 = \frac{P_1 - P_0}{t_{ot}} = \frac{P_1 \cdot S_{nom} - P_0 \cdot S_{nom}}{S_{nom} \cdot t_{ot}}.$$

If $P_0 = 0$, then $a_1 = P_1 / t_{ot} = 4/23 = 0.174$ mm/year,

$v_1 = 0.35$ (average value from Table 1),

$$Y = \frac{X_{\lim} - X_1}{a_1} = \frac{4,5}{0.174} = 25.86,$$

$$\tilde{\pi}_1 = Y \cdot Z = 25.86 \cdot 1.06 = 27.4 \text{ years.}$$

Table 1 — The coefficient of variation values for the main degradation (fracture) processes [8]

Type of degradation (degradation process)	Coefficient of variation for the degradation process
Mechanochemical wear	0.20 – 0.50

Example 2. Fatigue wear (crack formation)

After 23 years in service, let's estimate the residual life of a straight section of the pipeline. A total of 60 similar sections (the same ones from the previous example), all produced and put into operation at the same year, were subjected to the study. Measurements were conducted for pipes with a diameter of $D = 530 mm$ and a nominal wall thickness of $S_{nom} = 28\ mm$. During the operational period of time $t_{ot} = 23$ years, the measured crack opening width in the examined section's wall was $R_1 = 2 mm$, with a maximum permissible value of $R_{\lim} = 4\ mm$.

Calculating the residual life value in relative units of the initial data:

$$P_{lim} = R_{lim} = 4 \text{ mm},$$

$$P_1 = R_1 = 2 \text{ mm},$$

$v_2 = 0.51$ (average value from Table 2).

Let's normalize the data in relation to the thickness of the pipe wall $S_{nom} = 28\ mm$:

$$X_{\lim} - X_1 = \frac{P_{\lim}}{S_{nom}} - \frac{P_1}{S_{nom}} = \frac{P_{\lim} \cdot S_{nom} - P_1 \cdot S_{nom}}{S_{nom}} = \frac{4 \cdot 28 - 2 \cdot 28}{28} = 2.0,$$

$$Z = 1 + \frac{v_2^2}{2} = 1 + \frac{0.51^2}{2} = 1.13,$$

$$a_2 = \frac{P_1 - P_0}{t_{ot}} = \frac{P_1 \cdot S_{nom} - P_0 \cdot S_{nom}}{S_{nom} \cdot t_{ot}}.$$

If $P_0 = 0$, then $a_2 = P_1/t_{ot} = 2/23 = 0.087$ mm/year,

$$Y = \frac{X_{\lim} - X_1}{a_2} = \frac{2}{0.087} = 23,$$

$$\tilde{\pi}_2 = Y \cdot Z = 23 \cdot 1.13 = 26 \text{ years.}$$

Table 2 — The coefficient of variation values for operating times and degradation process types for typical thermomechanical equipment objects [8]

Object name	Primary types of degradation (failures)	Coefficient of variation
Pipelines	Aging, volumetric fatigue of welded joints and base metal	0.40 – 0.70

Example 3. Case of simultaneous occurrence of two degradation processes

The same 60 analogous pipeline sections, all produced in the same year, were subjected to the study after $t_{ot} = 23$ years of operation (the initial data is similar to examples 1 and 2). The pipes have a nominal wall thickness of $S_{nom} = 28 mm$.

1. Erosion-corrosion wear. During the operational period of time t_{ot}, the measured wall thickness of the examined section was $S_1 = 24$ mm, with a minimum permissible normative value of $S_p = 19.5 \, mm$.

2. Fatigue wear (crack formation). During the operational period of time t_{ot}, the measured crack opening width in the examined section's wall was $R_1 = 2 \, mm$, with a maximum permissible value of $R_{\lim} = 4 \, mm$.

3. Using data from the previous example, we introduce the following notations considering operating modes. According to the pipeline operation results in normal mode, it was found that the proportion of failures induced by the initial degradation process (erosion-corrosion wear) represents p_{01} of the total failures, while the second degradation process (fatigue wear) accounts for p_{02} failures, for example, $p_{01} = 0.8$ and $p_{02} = 0.2$. In this case $p_{01} + p_{02} = 1$.

4. During the operational period of time t_{ot} the average operating fluid temperature was $t_0 = 50 \, °C$.

Let's calculate the residual life of a straight pipeline section upon transitioning to an elevated operating fluid temperature of $t_1 = 70 \, °C$.

Solution. For further calculations, we will use the previously obtained values of $a_{01} = 0.174$ and $a_{02} = 0.087$, which were derived from normalized input data in relative units.

Calculating the coefficient of variation for the generalized degradation process using equation (7) for $v_{01} = 0.35$ and $v_{02} = 0.51$:

$$v_0 = \left(\sum_{j=1}^{m} \frac{v_{0j}^2 p_{0j}^2}{\sum_{j=1}^{m} p_{0j}^2} \right)^{1/2} = \left(\frac{0.35^2 \cdot 0.8^2}{0.8^2 + 0.2^2} + \frac{0.51^2 \cdot 0.2^2}{0.8^2 + 0.2^2} \right)^{1/2} = 0.36.$$

Calculating the average degradation rate:

$$a_0 = \left(\sum_{j=1}^{m} a_{0j}^2 \right)^{1/2} = \left(0.174^2 + 0.087^2 \right)^{1/2} = 0.194 \, mm \, / \, year,$$

$$Z = 1 + \frac{v_0^2}{2} = 1 + \frac{0.36^2}{2} = 1.065 \, .$$

Normalizing the initial data with respect to $S_{nom} = 28 \, mm$ and making calculations:

$$X_{lim.1} = \frac{P_{lim.1}}{S_{nom}} = \frac{8.5}{28} = 0.304 \, ,$$

$$X_{lim.2} = \frac{P_{lim.2}}{S_{nom}} = \frac{4}{28} = 0.143 \, .$$

Normalization of data allows us to calculate the average ($._a$) value:

$$X_{lim.a} = (X_{lim.1} + X_{lim.2})/2 = (0.304 + 0.143)/2 = 0.224,$$

$$X_{1.1} = \frac{P_{1.1}}{S_{nom}} = \frac{4.0}{28} = 0.143 \, ,$$

$$X_{1.2} = \frac{P_{1.2}}{S_{nom}} = \frac{2.0}{28} = 0.071 \, .$$

Calculating the average ($._a$) value:

$$X_{1.a} = (X_{1.1} + X_{1.2})/2 = (0.143 + 0.071)/2 = 0.107,$$

$$X_{lim.a} - X_{1.a} = \frac{P_{lim.a}}{S_{nom}} - \frac{P_{1.a}}{S_{nom}} \, ,$$

$$(X_{lim.a} - X_{1.a}) \cdot S_{nom} = P_{lim.a} - P_{1.a} = (0.224 - 0.107) \cdot 28 = 3.276.$$

Calculating the average residual life value:

$$\tilde{\pi}_3 = \frac{(P_{lim.a} - P_{1.a})}{a_0} \cdot Z = \frac{3.276}{0.194} \cdot 1.065 = 17.98 \, years.$$

Note 4. An increase in the working fluid temperature significantly reduces the pipeline's residual life, as it intensifies both degradation processes simultaneously.

This can be illustrated with the following example. The acceleration coefficients will be determined by performing calculations as the pipeline transitions from the normal operating mode to the accelerated one:

$$K_{f1j} = \exp\left[\frac{E_{aj}}{K} \left(\frac{1}{T_0} - \frac{1}{T_1} \right) \right].$$

$T_0 = t_0 °C + 273 = 50 + 273 = 323 °K;$
$T_1 = t_1 °C + 273 = 70 + 273 = 343 °K;$
$E_{a1} = 0.3$ eV which is the activation energy of the first degradation process (erosion-corrosion wear);
$E_{a2} = 0.5$ eV which is the activation energy of the second degradation process (fatigue wear);
$K = 8.6173 \cdot 10^{-5}$ eV·K^{-1};

$$K_{f11} = \exp\left[\frac{E_{a1}}{K}\left(\frac{1}{T_0} - \frac{1}{T_1}\right)\right] = \exp\left[\frac{0{,}3}{8.6173 \cdot 10^{-5}}\left(\frac{1}{50+273} - \frac{1}{70+273}\right)\right] = 1.8747,$$

$$K_{f12} = \exp\left[\frac{E_{a2}}{K}\left(\frac{1}{T_0} - \frac{1}{T_1}\right)\right] = \exp\left[\frac{0{,}5}{8.6173 \cdot 10^{-5}}\left(\frac{1}{50+273} - \frac{1}{70+273}\right)\right] = 2.8503.$$

Calculating degradation rates of each degradation process in the accelerated mode:

$$a_{11} = a_{01}K_{f11} = 0.174 \cdot 1.8747 = 0.326,$$

$$a_{12} = a_{02}K_{f12} = 0.087 \cdot 2.8503 = 0.2479.$$

Calculating failure proportions for each degradation process during the transition to the accelerated operating mode:

$$p_{1j} = \frac{p_{0j}K_{f1j}}{\sum\limits_{j=1}^{m} p_{0j}K_{f1j}}.$$

$$p_{11} = \frac{p_{01}K_{f11}}{\sum\limits_{j=1}^{m} p_{0j}K_{f1j}} = \frac{0.8 \cdot 1.8747}{(0.8 \cdot 1.8747) + (0.2 \cdot 2.8503)} = 0.7245,$$

$$p_{12} = \frac{p_{02}K_{f12}}{\sum\limits_{j=1}^{m} p_{0j}K_{f1j}} = \frac{0.2 \cdot 2.8503}{(0{,}8 \cdot 1.8747) + (0.2 \cdot 2.8503)} = 0.2754.$$

Determining the coefficient of variation for the generalized degradation process under the temperature-accelerated operational conditions:

$$V_1 = v_1 = \left(\sum\limits_{j=1}^{m} \frac{v_{0j}^2 p_{1j}^2}{\sum\limits_{j=1}^{m} p_{1j}^2}\right)^{1/2} = \sqrt{\frac{0.35^2 \cdot 0.7245^2}{0.7245^2 + 0.2754^2} + \frac{0.51^2 \cdot 0.2754^2}{0.7245^2 + 0.2754^2}} = 0.3739.$$

Calculating the average rate of the pipeline's generalized degradation process in the accelerated operating mode:

$$a_1 = \left(\sum\limits_{j=1}^{m} a_{1j}^2\right)^{1/2} = \sqrt{0.326^2 + 0.2479^2} = 0.4095.$$

Calculating: $Z = 1 + (v_1^2 / 2) = 1 + (0.3739^2 / 2) = 1.07$.

Calculating average residual life value considering variable operating modes:

$(X_{lim.a} - X_{1.a}) \cdot S_{nom} = P_{lim.a} - P_{1.a} = (0.224 - 0.107) \cdot 28 = 3.276$ (see the previous example),

$$\tilde{\pi}_4 = \frac{(P_{lim.a} - P_{1.a})}{a_1} \cdot Z = \frac{3.276}{0.4095} = 8.56 \; years.$$

3. Conclusions

All the methods presented in the article for assessing residual life and based on studying the dynamics of a single determining (resource) parameter ($\widetilde{\pi}_1$=27.4 years and $\widetilde{\pi}_2$=26 years) yielded results of a similar order of magnitude. The most adequate method should be considered the one that takes into account the simultaneous occurrence of several (two) degradation processes in the product ($\widetilde{\pi}_3$ =17.98 years). The method involving the assessment of $\widetilde{\pi}_3$ leads to the most reliable result, based on the most general considerations that require no proof, and is recommended in cases where there is initial data on the degradation processes occurring in the product.

The paper also introduces a new method that, based on the known fractional contribution of constituent degradation processes in normal operating mode, enables the assessment of diffusion distribution parameters in accelerated mode. Based on these parameters, it enables correct calculations of the product's residual life at elevated operating temperatures ($\widetilde{\pi}_4$=8.56 years).

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