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VARIATIONAL ASSIMILATION OF CONCENTRATION MEASUREMENTS IN RADIONUCLIDE TRANSPORT MODEL IN A MARINE ENVIRONMENT

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Анотація. *У роботі розроблено варіаційний алгоритм асиміляції даних у моделі переносу радіонуклідів, який дозволяє ідентифікувати розподілене джерело радіоактивності після атмосферних випадінь на поверхню моря. Алгоритм включає розв'язання спряжених рівнянь тривимірної моделі транспорту радіонуклідів у морському середовищі ТРИТОКС для оцінки елементів матриці джерело-рецептор (МДР). Кількість інтегрувань спряжених рівнянь ТРИТОКС, необхідна для повного розрахунку МДР, дорівнює кількості вимірювань. Квадратична функція витрат, яку необхідно мінімізувати в алгоритмі, характеризує відхилення розрахованих та спостережуваних концентрацій, а також відмінність першого наближення поля випадіння від отриманого розв'язку. Коваріаційна матриця помилок моделі, що входить до функції витрат, була параметризована за допомогою аналітичних виразів. Функцію витрат було мінімізовано аналітичним методом, який не гарантує позитивності рішення. Однак у чисельних експериментах в даній роботі розв'язок залишався додатнім. Метод було перевірено на даних штучних вимірів концентрацій цезію-137, розрахованих для задачі розповсюдження радіонуклідів у Чорному морі після аварії на Чорнобильській АЕС. «Істинне» поле осадження вважалося рівним результату інтерполяції вимірювань у верхньому шарі води, зібраних у червні 1986 року, на розрахунковій сітці та нормалізованим так, щоб загальне осадження на поверхню Чорного моря дорівнювало 2 ПБк. Оскільки поля течій у Чорному морі за 1986 рік були відсутні, у цьому дослідженні використовувалися кліматологічні щомісячні усереднені поля течій за період 2005–2015 рр. Як перше наближення використовувалась постійна щільність осадження. Оцінене в результаті асиміляції даних поле осадження було близьким до істинного. Результати представленого попереднього тестування алгоритму показали його потенціал для практичного використання.*
Ключові слова: *асиміляція даних, варіаційний метод, Чорнобильська аварія, Чорне море, ТРИТОКС.*

Abstract. *In this paper, a variational data assimilation algorithm within the radionuclide transport model has been developed, which enables the identification of a distributed source of radioactivity following atmospheric fallout on the sea surface. The algorithm involves the solution of the adjoint equations of a three-dimensional model of radionuclide dispersion in the marine environment, THREETOX, to evaluate the elements of the source-receptor matrix (SRM). The number of adjoint THREETOX code integrations required for the full SRM calculation is equal to the number of measurements. The quadratic cost function to be minimized in the algorithm characterizes the deviation of simulated and observed concentrations, together with the difference between the first guess deposition field and the estimated solution. The covariance matrix of model errors, entering the cost function, has been parameterized using analytical expressions. The cost function has been minimized analytically, but it does not guarantee the positivity of the solution. However, the solution remained positive in numerical experiments in this work. The method has been tested against artificial measurements of caesium-137 concentrations, calculated for the problem of radionuclide dispersion in the Black Sea following the Chernobyl accident. The «true» deposition field*

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has been considered equal to the results of interpolation of measurements in the upper water layer, collected in June 1986, on the model grid and normalized in such a way that total deposition on the surface of the Black Sea equaled 2PBq. Since the fields of currents in the Black Sea for 1986 were not available, the climatological monthly averaged fields of currents for the period 2005–2015 have been used in this study. The first guess estimation of the deposition density field has been set to a constant value at the surface of the Black Sea. The estimated deposition field by data assimilation closely matched the true field. The results of the presented preliminary testing of the algorithm have shown its potential for use in practical applications.
Keywords: *data assimilation, variational method, inverse problem, Chernobyl accident, Black Sea, THREETOX.*

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1. Introduction

One of the key problems in the marine dispersion modelling is the determination of radionuclide sources. Due to the FDNPP accident, there are several major sources of radionuclide contamination to the marine environment: (i) atmospheric deposition of radionuclides onto the sea surface, (ii) direct release of radionuclides to the ocean, (iii) releases from land via river and coastal runoff, (iv) groundwater release. The feasible way for determining the source term is to combine radionuclide measurement data and dispersion models («inverse modeling»). A number of atmospheric dispersion models using different tracer inversion algorithms were used to estimate deposition onto the ocean surface after the Fukushima Daiichi Nuclear Power Plant accident [1]. The direct release scenarios [2, 3] were constructed using monitoring data in the vicinity of FDNPP to scale computations. In the work [4], a total direct release of 5.1-5.5 PBq was estimated using a four-step inverse approach based on the measured 137Cs activity in the south and north outlet channels of FDNPP. The estimate of direct release in the work [5] was based on interpolated monitoring data in a 50-km area around FDNPP, and the environmental half-time, deduced from observations. The authors of the work [6] estimated the source term using inverse estimation of direct releases based on the Green function approach [7]. The inversion method based on minimizing the model-cruise data difference was applied in the work [8] to estimate releases. However, methods of inverse modeling were rarely applied to the identification of the distributed source due to the atmospheric deposition of radionuclides.

The current paper aims to develop an effective algorithm of data assimilation in the radionuclide transport model to identify a distributed source.

2. Method description

2.1. Transport model

In this article, we consider a simplified transport model in which the processes of radionuclides are considered tracers and physical processes such as absorption, sedimentation, and decay are neglected. This is a reasonable assumption in many cases, such as the problem considered below of Cs-137 transport in the Black Sea a few years after the Chernobyl accident. The data assimilation method, however, is directly extendable to more general cases.

Hence, the transport of radionuclide concentration c is described by the following equation applicable to every passive scalar [9, 10]:

$$\frac{\partial Dc}{\partial t}+\frac{\partial Duc}{\partial x}+\frac{\partial Dvc}{\partial y}+\frac{\partial \omega c}{\partial \sigma}-\frac{\partial}{\partial x}\left(K_h D\frac{\partial c}{\partial x}\right)-\frac{\partial}{\partial y}\left(K_h D\frac{\partial c}{\partial y}\right)-\frac{\partial}{\partial \sigma}\frac{K_v}{D}\frac{\partial c}{\partial \sigma}=q_s . \qquad (1)$$

Here, the terrain following sigma coordinate system is used, which is Cartesian in the horizontal (x,y) plane. The vertical coordinate σ is defined as

$$\sigma = \frac{z+\eta}{H+\eta} = \frac{z+\eta}{D},$$

where $H = H(x,y)$ is the depth of the water body with an undisturbed free surface and $\eta = \eta(x,y)$ is the deviation of the free surface from the undisturbed level.

The meaning of other variables in (1) is the following: u, v are velocity components along axes *x* and *y* respectively; ω is an analog of vertical velocity in the sigma-coordinate system [9]; K_h, K_v are horizontal and vertical turbulent diffusivities; and q_s is the volumetric density of sources. Equation (1) is complemented with initial and boundary conditions:

$$c(x,y,\sigma,0) = 0, \tag{2a}$$

$$\frac{\partial c(x,y,\sigma,t)}{\partial \overline{n}} = 0,\ \forall (x,y,z,t) \in \partial\Omega \times [0,T], \tag{2b}$$

where $\Omega \subset R^3$ is the integration domain (water body) and $\partial\Omega$ is the boundary of the domain, $\overline{n}$ is normal to the boundary, *T* is the length of the time interval for which equation (1) is solved.

For the following material, it is convenient to represent Eq. (1) in a shortened form:

$$Lc = q_s \tag{3a}$$

with linear operator *L*:

$$L \equiv \frac{\partial D}{\partial t} + \frac{\partial Du}{\partial x} + \frac{\partial Dv}{\partial y} + \frac{\partial \omega}{\partial \sigma} - \frac{\partial}{\partial x} K_h D \frac{\partial}{\partial x} - \frac{\partial}{\partial y} K_h D \frac{\partial}{\partial y} - \frac{\partial}{\partial \sigma} \frac{K_v}{D} \frac{\partial}{\partial \sigma}, \tag{3b}$$

defined on a subset of functions integrable with a square in the spatio-temporal domain $\Omega \times (0,T)$ and satisfying the initial and boundary conditions (2).

Below, a particular case of q_s, which represents instantaneous deposition of radioactivity at $t = 0$, is considered:

$$q_s(x,y,z,t) = Q_s(x,y)\,\delta_\varepsilon(t-0)\,\delta_{\varepsilon_1}(\sigma - 0). \tag{4}$$

Here, $Q_s(x,y)$ is the initial deposition field; the function of the scalar argument $\delta_\varepsilon(.)$ in Eq. (4) is a stepwise function:

$$\delta_\varepsilon(\tau) = \begin{cases} 1/\varepsilon, & |\tau| \le \varepsilon \\ 0, & |\tau| > \varepsilon \end{cases}. \tag{5}$$

Here, the parameter ε is sufficiently small (much smaller than the temporal scale of the considered atmospheric dispersion problem) so that, from a physical point of view, the source could be considered as a point, and as a result, the obtained solution does not depend on ε. The function $\delta_{\varepsilon_1}(.)$ is defined analogously. We do not use the Dirac delta function because even though Eq. (1) with the Dirac delta function on the r.h.s has a solution in the space of generalized functions, the scalar product is not defined in this space, and thus the presentation of the adjoint formalism below will not be mathematically correct. Note that following the numerical discretization of Eq. (1), the values of $\varepsilon, \varepsilon_1$ are defined as: $\varepsilon = \delta t$, $\varepsilon_1 = \delta\sigma$, where δt and $\delta\sigma$ are correspondingly the time step of model integration and the grid size in the vertical direction.

The numerical approximation and solution of equation (1) is described in [11] and thus it is not discussed here.

2.2. Statement of data assimilation problem

In this work, we consider the estimation of the r.h.s. of Eq. (1), source term q_s, and thus initial deposition field $Q_s(x,y)$ with the aid of measurements and a transport model. Thus, we solve the inverse problem, which, following the Bayesian formulation [12], could be posed as a problem of minimizing the following cost function:

$$J = \left(\bar{\xi} - \underline{\underline{G}}\bar{q}\right)^T \underline{\underline{F}}^{-1} \left(\bar{\xi} - \underline{\underline{G}}\bar{q}\right) + \left(\bar{q}_0 - \bar{q}\right)^T \underline{\underline{B}}^{-1} \left(\bar{q}_0 - \bar{q}\right). \tag{6}$$

Here, $\bar{\xi} \in R^{N_O}$ is a vector of measurements with N_O being a total number of measurement values; $\bar{q} \in R^{N_G}$ is the control vector, which in this case consists of deposition values to be estimated in grid nodes of the model, N_G being a number of computational grid nodes in which deposition is to be estimated; $\bar{q}_0 \in R^{N_G}$ is the first guess (prior) estimation of deposition; $\underline{\underline{B}} \in R^{N_G \times N_G}$ is covariance matrix of the assumed errors of the prior estimation of deposition vector; matrix $\underline{\underline{F}} = \underline{\underline{O}} + \underline{\underline{M}}$ where matrix $\underline{\underline{O}} \in R^{N_O \times N_O}$ is covariance matrix of the measurement errors, $\underline{\underline{M}} \in R^{N_O \times N_O}$ is covariance matrix of model errors estimated under condition of exactly known control vector.

Matrix $\underline{\underline{G}} \in R^{N_O \times N_G}$ entering eq. (6) relates deposition in the vector $\bar{q}$ to the simulated concentrations by the model at times and locations of the corresponding measurements. This matrix, which is also called "source-receptor matrix" (SRM), is to be precalculated by the model as described in the next paragraph.

2.3. Source-receptor matrix

Typically, there are two main approaches for the calculation of the source-receptor matrix $\underline{\underline{G}}$. In the first approach, model integration is to be performed as many times as there are grid nodes at which deposition is defined. Each i-th run is performed with unit deposition set in the single i-th node. The output of the i-th run at times and locations of measurements gives the i-th column of the matrix $\underline{\underline{G}}$. This approach requires N_G model integrations.

In the case when the size of the control vector is much greater than the number of measurements $N_G >> N_O$, the calculation of the SRM could be performed with only N_O model integrations of adjoint equations [13]. The adjoint operator to the operator L is defined by the Lagrange duality relationship

$$\left(L\varphi, \gamma\right) = \left(\varphi, L^*\gamma\right), \tag{7}$$

which should be valid for arbitrary functions φ and γ, defined and square-integrable on Ω, and meet the boundary conditions (2b). Here, the notation of the scalar product is used:

$$\int_0^T \int_\Omega \chi \cdot \gamma \cdot d\Omega \cdot dt = \left(\chi, \gamma\right). \tag{8}$$

The following definition for the adjoint operator to the operator L in (3b) can be verified using the same derivation procedure as presented in [14]:

$$L^* \equiv -D\frac{\partial}{\partial t} - Du\frac{\partial}{\partial x} - Dv\frac{\partial}{\partial y} - \omega\frac{\partial}{\partial \sigma} - \frac{\partial}{\partial x}K_h D\frac{\partial}{\partial x} - \frac{\partial}{\partial y}K_h D\frac{\partial}{\partial y} - \frac{\partial}{\partial \sigma}\frac{K_v}{D}\frac{\partial}{\partial \sigma}, \tag{9}$$

which acts on a subset of functions integrable with squares and satisfying the following boundary conditions and initial conditions:

$$c^*\left(x, y, \sigma, \mathrm{T}\right) = 0\,, \tag{10a}$$

$$\frac{\partial c^*\left(x, y, \sigma, t\right)}{\partial \overline{n}} = 0,\ \forall\left(x, y, z, t\right) \in \partial\Omega \times \left[0, T\right]. \tag{10b}$$

Now, let us define the measurement equation, which relates the calculated concentration to the j-th measurement:

$$c_j = \int_0^T \int_\Omega c \cdot p_j \cdot d\Omega \cdot dt = \left(c, p_j\right). \tag{11}$$

The calculated concentration corresponding to the j-th measurement is thus the scalar product of the concentration field and the «probing function» p_j which can be defined as

$$p_j\left(x, y, z, t\right) = \delta_\varepsilon\left(x - x_j^o\right) \cdot \delta_\varepsilon\left(y - y_j^o\right) \cdot \delta_\varepsilon\left(\sigma - \sigma_j^o\right) \cdot \delta_{\varepsilon_1}\left(t - t_j^o\right). \tag{12}$$

Here, the index «o» denotes coordinates and measurement time. Functions $\delta_\varepsilon\left(.\right), \delta_{\varepsilon 1}\left(.\right)$ are defined as in Eq.(5), and parameters ε, ε_1 are sufficiently small to represent a measurement at a given point, so that their particular values are of no importance.

Now, let us define the adjoint variable c_j^* as the solution of the following adjoint equation:

$$L^* c_j^* = p_j\,, \tag{13}$$

which, according to Eq. (10a) is integrated backwards in time starting from $t = T$. Following the definition of an adjoint operator and duality relationship, the solution of Eq. (13) satisfies the Lagrange duality relationship (7): $\left(Lc, c_j^*\right) = \left(L^* c_j^*, c\right)$ and hence

$$\left(q^s, c_j^*\right) = \left(p_j, c\right) = c_j\,. \tag{14}$$

The solution of the adjoint equation (13) $c_j^*\left(x, y, 0, 0\right)$, discretized at grid nodes of the computational domain, defines the j-th row of the matrix $\underline{\underline{G}}$. Note that the above statement follows from the particular form of the source function q_s in Eq. (4), which is nonzero only in the small vicinity of $\sigma = 0$, and $t = 0$. Finally, the full computation of the source-receptor matrix $\underline{\underline{G}}$ requires N_O integrations of adjoint equations (14).

2.4. Parameterization of covariance matrices and inverse problem solution

Apart from SRM, other matrices entering the cost function (6) need to be defined as well. Matrix $\underline{\underline{F}}$ representing the combined error of measurements and model estimations in measurement points is assumed diagonal, which is a reasonable assumption provided that measurements are scarce in time and in space:

$$\begin{aligned} &\underline{\underline{F}} = diag\{\sigma_{Fi}^2\},\ 1 \le i \le N_O \\ &\sigma_{Fi}^2 = \sigma_{Oi}^2 + \sigma_{Mi}^2 \end{aligned}\ . \tag{15}$$

The first-guess estimation is defined in all computational grid nodes and thus off-diagonal terms in background error covariance matrix $\underline{\underline{B}}$ cannot be neglected. Therefore, the following approximation is used for the elements of the covariance matrix b_{nl} representing the covariance of error of first guess estimation in grid nodes n and l [15]:

$$b_{nl} = \sigma_{Bn}\sigma_{Bl} \exp\{-r_{nl}^2 / R_0^2\}. \tag{16}$$

Here, σ_{Bn}^2 is the mean squared error of the first guess estimation in the node n, and R_0 is the correlation radius. Thus, the covariance function is assumed isotropic and depends only on the distance between grid nodes.

The minimum of the cost function J defined in (6) can be found by taking the derivative with respect to $\overline{q}$ and equating it to zero. Since the cost function is convex, the stationary point defines a minimum, which is a unique solution. This leads to the following system of equations:

$$\left(\underline{\underline{B}}\,\underline{\underline{G}}^T \underline{\underline{F}}^{-1} \underline{\underline{G}} + \underline{\underline{I}}\right)\overline{q} = \underline{\underline{B}}\,\underline{\underline{G}}^T \underline{\underline{F}}^{-1} \overline{\xi} + \overline{q}_0, \tag{17}$$

where $\underline{\underline{I}}$ is the identity matrix. System (17) is solved against $\overline{q}$ which is the solution of the minimization problem, and in this case, defines the estimated initial deposition.

Note that there is no guarantee that the solution of the equation (17) will be positive. If negative values of the elements of the vector $\overline{q}$ appear after the solution of (17), then another approach based on iterative minimization of the cost function subject to restriction $\overline{q} \geq 0$ should be used. However, in this work, such situations did not happen.

2.5. Implementation details

The solutions to adjoint equations (9), (10) and (12) have been implemented in the THREETOX code [11]. Numerical approximation of the adjoint operator (9) is performed with the same numerical schemes as used for approximation of the forward equations (1). To achieve this, velocity components are simply inverted ($\tilde{u} = -u$, $\tilde{v} = -v$, $\tilde{\omega} = -\omega$) and the same subroutines used for numerical approximations of operator L are applied for numerical approximations of the adjoint operator. The only difference is that since the adjoint operator is derived in non-divergent form, and if divergence $Div(u,v,\omega) = \frac{\partial Du}{\partial x} + \frac{\partial Dv}{\partial y} + \frac{\partial \omega}{\partial \sigma}$ is for some reason non-zero (e.g., because of numerical errors in the solution of the forward problem), then after the approximation of the operator L^* in a non-divergent form, the term $c^* \cdot Div(\tilde{u}, \tilde{v}, \tilde{\omega})$ need to be subtracted to obtain the most exact numerical approximation of adjoint operator to the forward problem.

The adjoint equation (13) is solved separately for each j-th measurement which defines the right part p_j, and the obtained solution c_j^* is stored in a binary file. After solving all the adjoint equations corresponding to each measurement, the binary files are read to construct the source-receptor matrix $\underline{\underline{G}}$ and to solve the equation (17).

3. Results of simulations

3.1. Model setup for the simulation of radionuclide dispersion in the Black Sea after the Chernobyl accident

In this paper, we have studied the possibility of evaluating surface deposition in the Black Sea region after the Chernobyl accident using post-accident measurements in June and October 1986 [16] and a variational data assimilation method described in the previous section. The locations of the measurements are shown in Fig. 1, whereas the fields of surface concentration of ^{137}Cs interpolated from measurements are shown in Fig. 2.

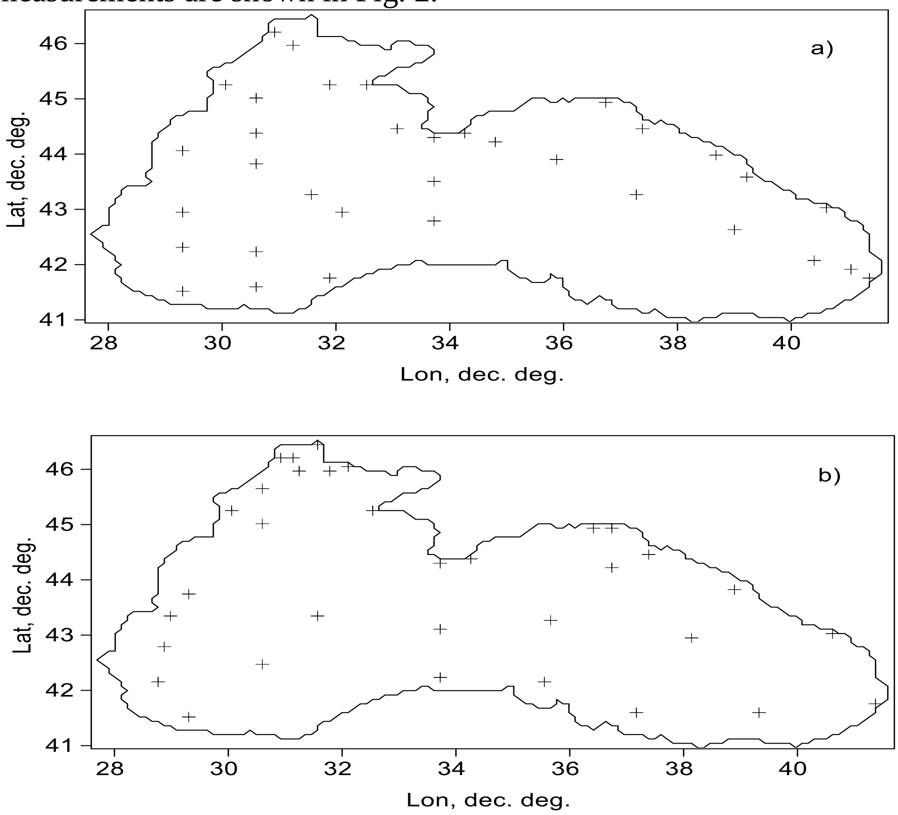

Figure 1 — Measurement locations in (a) June 1986, (b) October 1986

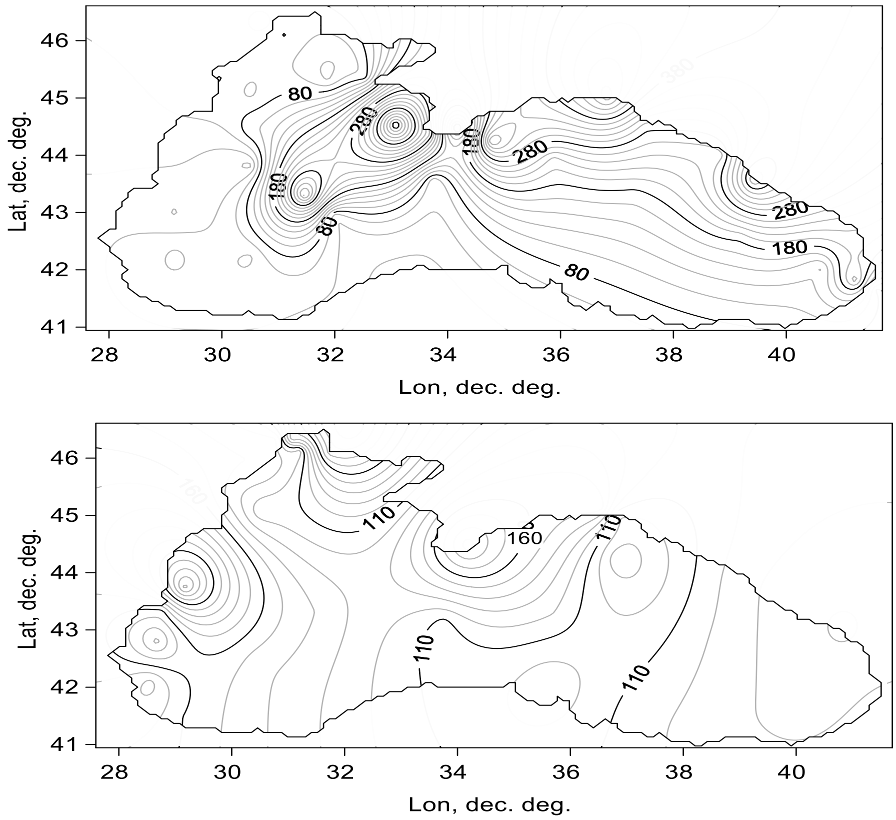

Figure 2 — Concentrations of Cs-137 obtained by measurements interpolation on model grid with kriging method; upper — June 1986, interval between isolines 20 Bq/m^3, maximum value 490 Bq/m^3; bottom — October 1986, interval between isolines 10 Bq/m^3, maximum value 250 Bq/m^3

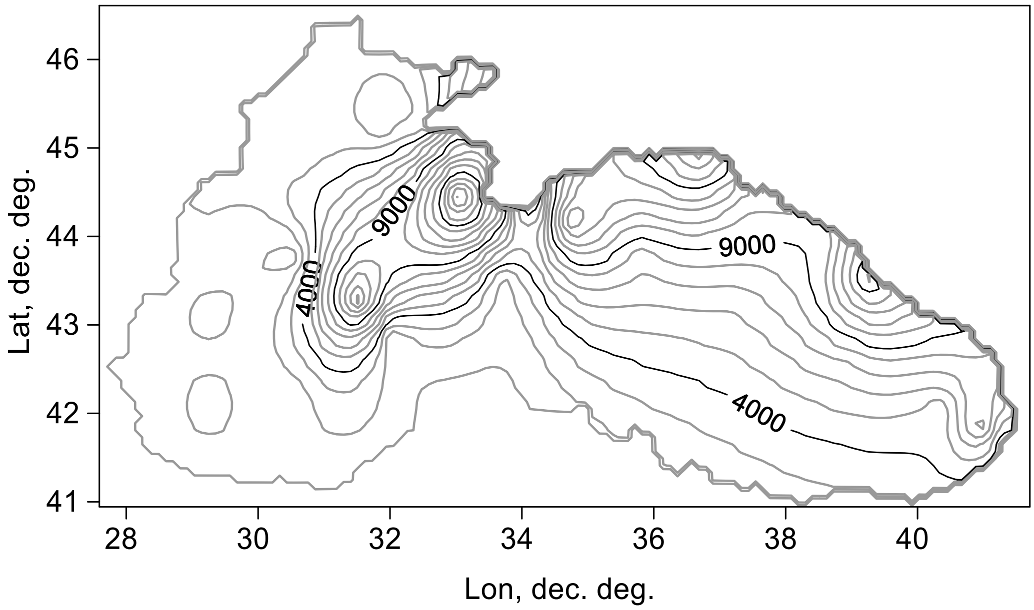

Figure 3 — Initial deposition of Cs-137 considered as «true» and used in the forward run; the isolines are drawn with a step of 1000 Bq/m^2

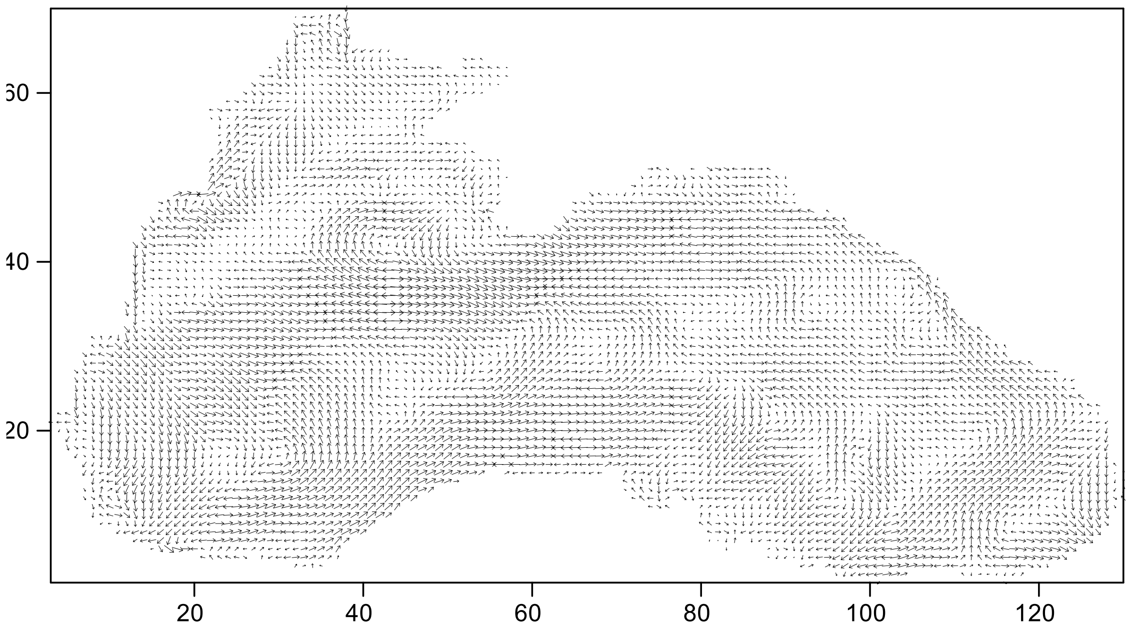

Figure 4 — Currents at the sea surface in May 1986

In our work, we have not yet used real measurement values because we first intended to test the data assimilation method in a situation of a nearly «perfect» forward model, when errors of the forward model as compared to real measurements do not interfere with the errors introduced by the process of source inversion. To do that, we have used «identical twin» numerical experiments [15] in which simulated values are used as «synthetic» measurements in the process of data assimilation.

Therefore, we first performed a forward run from which the «synthetic» measurements were extracted using the following approach. The time history of deposition after the Chernobyl accident was neglected. Instead, it has been assumed that deposition was instantaneous and occurred on May 10th, 1986. The «true» deposition field has been calculated by interpolation of concentration measurements collected in June 1986 on the model grid using the kriging method. The obtained concentration field has been multiplied by a constant factor to obtain the deposition field (Fig. 3) with the total amount of deposition equal to 2 PBq [17]. The obtained concentration and deposition fields were very close to those presented in [17].

Since the information about currents in the Black Sea for 1986 was not available, we used climatological (monthly averaged) fields of currents obtained from [18] and averaged for the period 2005–2015. The currents have been interpolated on the model grid, which covered the volume of the Black Sea with about 8500 m horizontal resolution and with 30 vertical levels. An example of a velocity field near the surface obtained for May is shown in Fig. 4.

The surface level was kept constant during each month. The vertical velocity has been calculated using the method described in [19] by solving the equation for each vertical column:

$$\frac{\partial^2 \omega}{\partial \sigma^2} = -\frac{\partial}{\partial \sigma}\left(\frac{\partial Du}{\partial x} + \frac{\partial Dv}{\partial y}\right). \qquad (18)$$
$$\omega\big|_{\sigma=0} = \omega\big|_{\sigma=1} = 0$$

3.2. Data assimilation method testing

As described in the previous section, at the first stage of the data assimilation method, the adjoint equations are solved, and the source-receptor matrix is obtained. Therefore, the accuracy of the calculated SRM needs to be tested by comparing concentrations calculated in the forward run and by multiplying the SRM by the true vector of initial depositions. The corresponding comparison is shown in Fig. 5. The correlation of true values and those obtained with the SRM is high and equal to 98.9 %. The mean relative error of values calculated with the SRM is 4.5 %, which is typically much less than the errors of the model as compared to real measurements. Therefore, the SRM can be robustly used instead of running a forward model in the source inversion procedure.

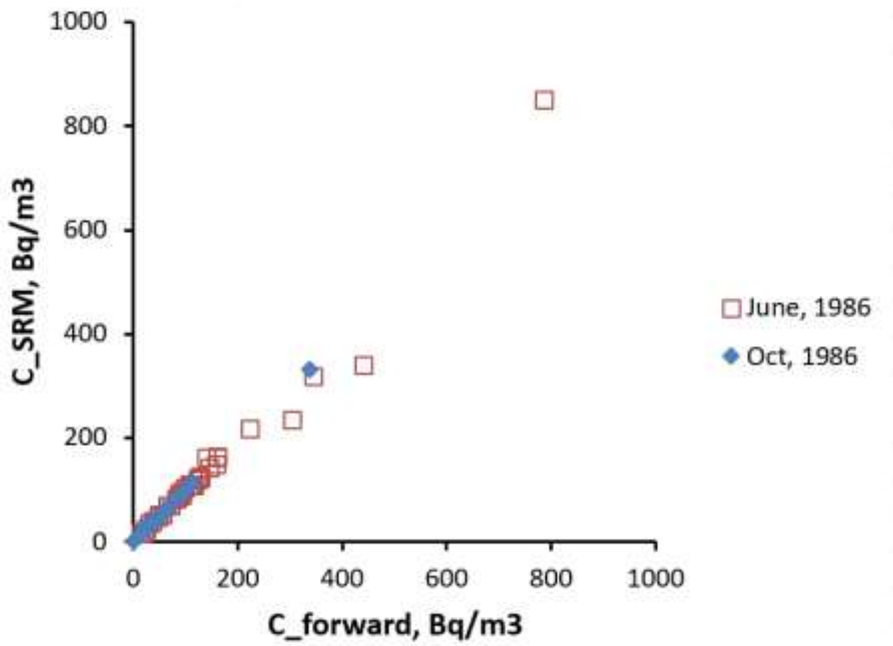

Figure 5 — Comparison of the concentrations calculated in the forward run and with the SRM obtained as a result of the adjoint equations solution

The next step included the use of the calculated SRM in the minimization procedure (17) to obtain the initial deposition field. At this stage, the initial deposition has been assumed unknown, and the first guess estimation of initial deposition has been set constant in the Black Sea waters and equal to 4000 Bq/m^2. The integral of this value over the territory of the Black Sea yielded the realistic amount of the deposited radioactivity close to a true run (1.6E15 Bq).

The diagonal elements of matrix $\underline{\underline{F}}$ was a sum of the mean squared errors of the model and observations σ_F^2 have been set equal to 5% of the observed value: $\sigma_{F,i} = 0.05\xi_i$, which is consistent with the presented above values of mean relative error resulting from the SRM application. The covariance matrix parameters of the first guess estimation have been set as follows. Root mean squared error of the first guess estimation has been set constant in all grid nodes and equal to $\sigma_B = 6000$ Bq/m^2. In the problems of source term estimation following accidental release of radioactivity, the assumed error of the first guess estimation of release rate is typically set by a factor from 1 to 10 of the mean value of the first guess estimation [20, 21].

The correlation radius R_0 of the errors in the first guess estimation has been set equal to 140 km. This is a typical value used in atmospheric data assimilation systems for scalars such as surface temperature and humidity [22, 23]. Since initial deposition is created by atmospheric fallout, it is reasonable to assume that at a sufficient distance from the source, when the cloud is well mixed, the correlation radius of the first guess deposition errors is similar to that of the other scalars in the atmosphere.

The deposition obtained by the solution of the inverse problem is shown in Fig. 6. The calculated deposition is qualitatively close to the true one. This is also confirmed by the correlation of the calculated and true deposition, which is equal to 91%. It can also be seen that the estimated deposition is smoothed as compared to the true deposition — estimated maximums in deposition are lower than in the true field. As shown in [23], this smoothing is a general feature of the linear regression algorithm expressed by the cost function (6).

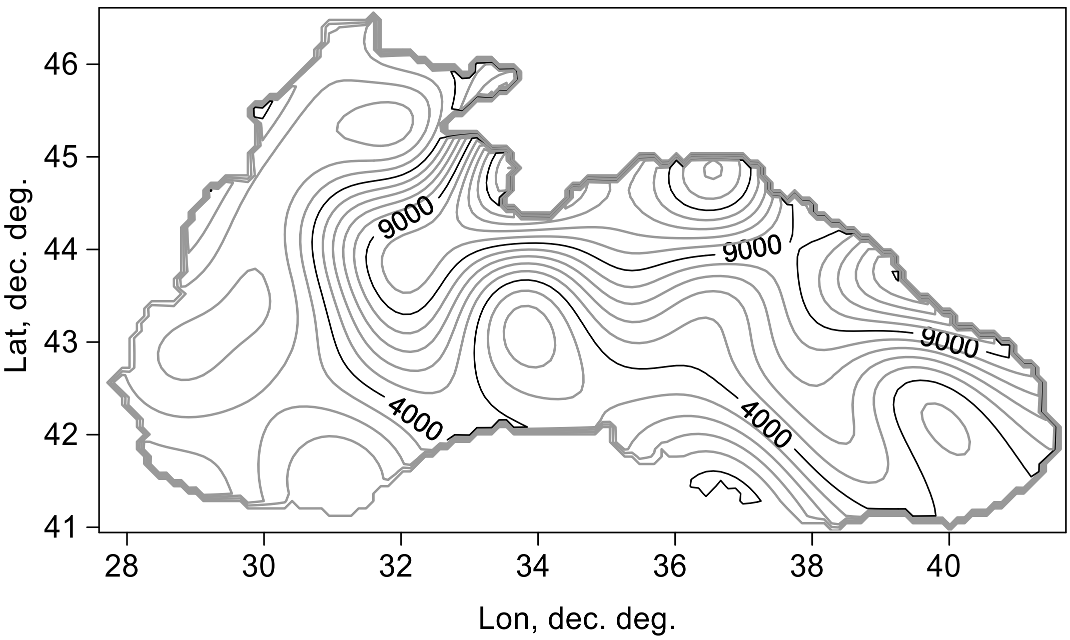

Figure 6 — Deposition of Cs-137 estimated as a result of the solution of the inverse problem; the isolines are drawn with a step of 1000 Bq/m^2

4. Conclusions

In this work, the variational algorithm of data assimilation within the radionuclide transport model has been developed, which enables the identification of a distributed source of radioactivity following atmospheric fallout on the sea surface. The algorithm involves the solution of the adjoint equations of a three-dimensional model of radionuclide dispersion in the marine environment, THREETOX, to evaluate the elements of the source-receptor matrix (SRM). The number of integrations of the adjoint THREETOX code, required for the full SRM calculation, equals the number of measurements. The quadratic cost function to be minimized in the algorithm characterizes the deviation of simulated and observed concentrations, together with the difference between the first guess deposition field and the estimated solution. The covariance matrix of model errors, entering the cost function, has been parameterized using analytical expressions. The cost function has been minimized by an analytical method, which does not guarantee the positivity of the solution. However, with the reasonably good first guess approximation, the solution remained positive. The method has been tested against artificial measurements of Caesium-137 concentrations, calculated for the problem of radionuclide dispersion in the Black Sea following the Chernobyl accident. The «true» deposition field has been considered equal to the results of interpolation of measurements in the upper water layer, collected in June 1986, on the model grid and normalized in such a way that total deposition on the Black Sea surface equaled 2PBq. Because the fields of currents in the Black Sea for 1986 are not available, the climatological monthly averaged fields of currents for the period 2005–2015 have been used in this study. The first guess estimation of the deposition density field has been set to a constant value at the surface of the Black Sea. The estimated deposition field by data assimilation has been close to the true one. The results of the presented preliminary testing of the algorithm have shown its potential for use in practical applications.

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